GPS Theory and Applications for Vehicle Dynamics Estimation

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Dynamic Design Laboratory

Motivation

- There are many interesting applications for GPS technology
  - Vehicle dynamics and state estimation
  - Precision mapping for navigation and control
  - Racing: full state information at every point on the track
- The data-sheets can be difficult to understand
  - Once you understand how GPS functions, the data sheets become more clear

Motivation

- There are a few lessons we have learned working with GPS, for instance
  - All velocities are not the same
  - All receivers are not created equal
  - The PPS signal is very important for dynamics applications
- GPS systems can be expensive
  - It is important to know what you are paying for

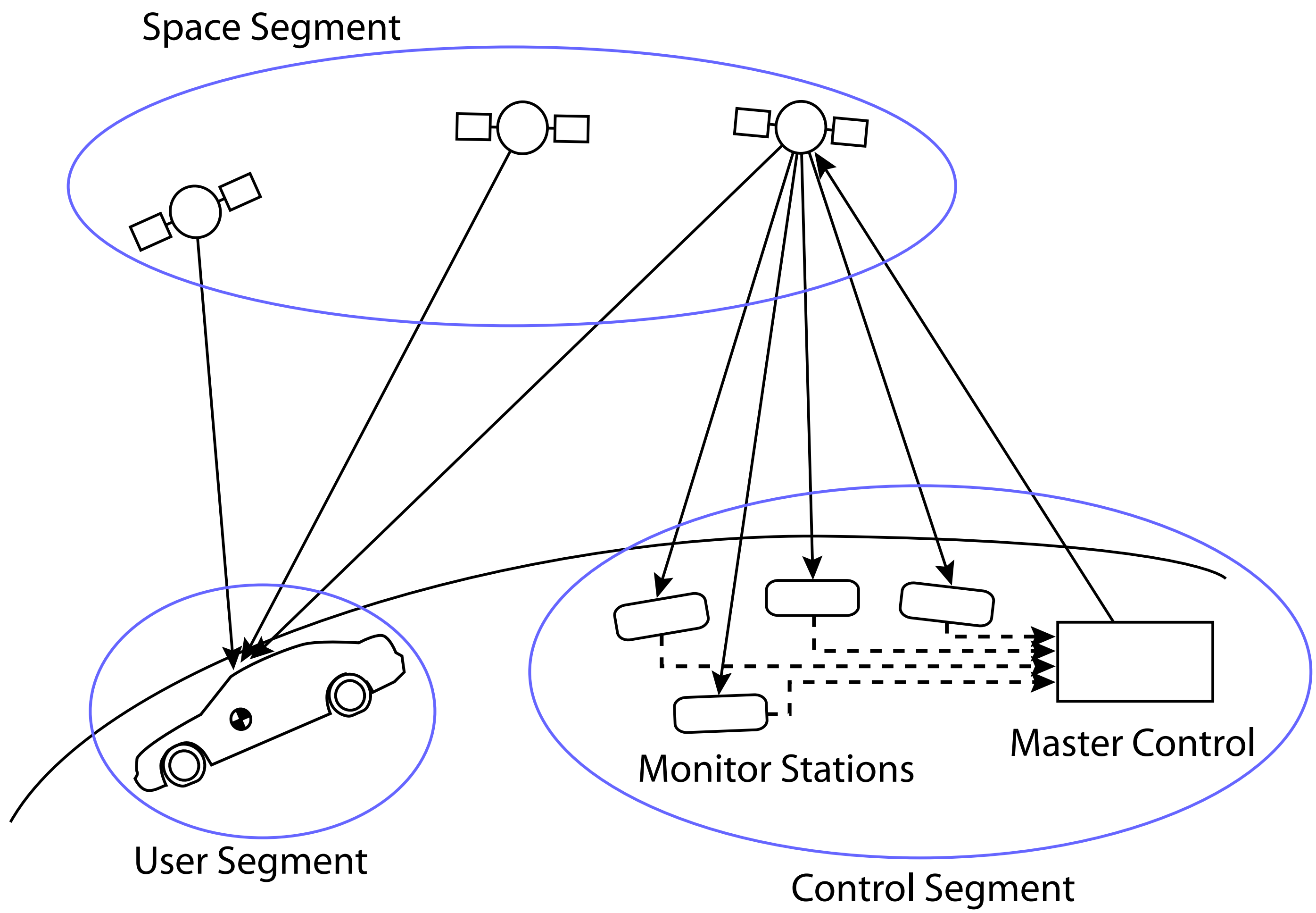
Goal of this Talk

- Show the important aspects of GPS theory for application to dynamic systems
- By the end of this talk you should be able to
  - Explain why the minimum number of satellites required by GPS is 4
  - Explain why the PPS signal is critical for dynamics applications
  - Interpret the important aspects of a GPS data sheet for your application
- Present the GPS data taken on the Technoshuttle

Overview

- GPS Big Picture
- GPS Signal Processing
- Error Sources
- Differential GPS
- GPS Attitude Determination
- Example Data Sheets from Industry
- Summary
- GPS-INS Based Vehicle Dynamics Estimation

Three GPS Segments

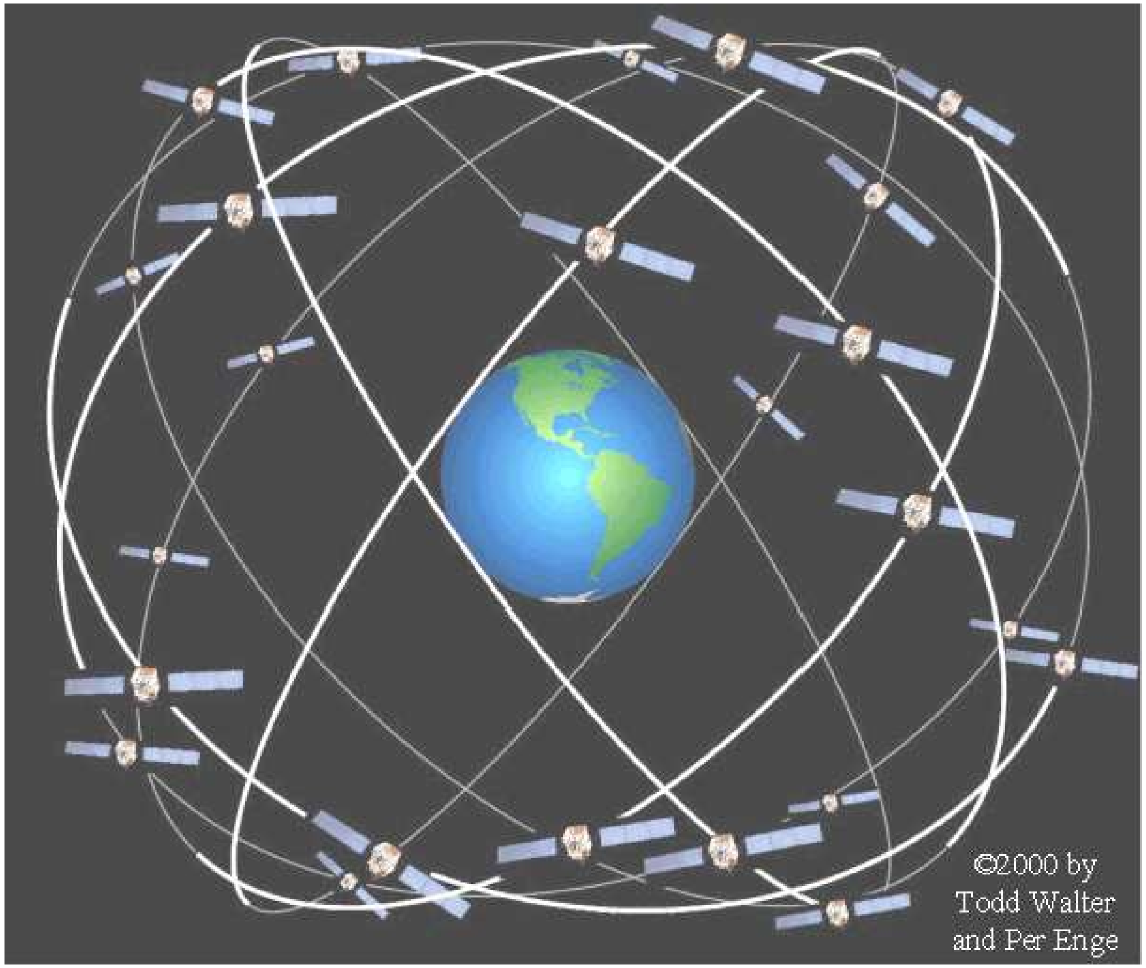


- Control, Space and User Segment

Control Segment

- Control segment tracks satellites and updates their orbit information

Space Segment

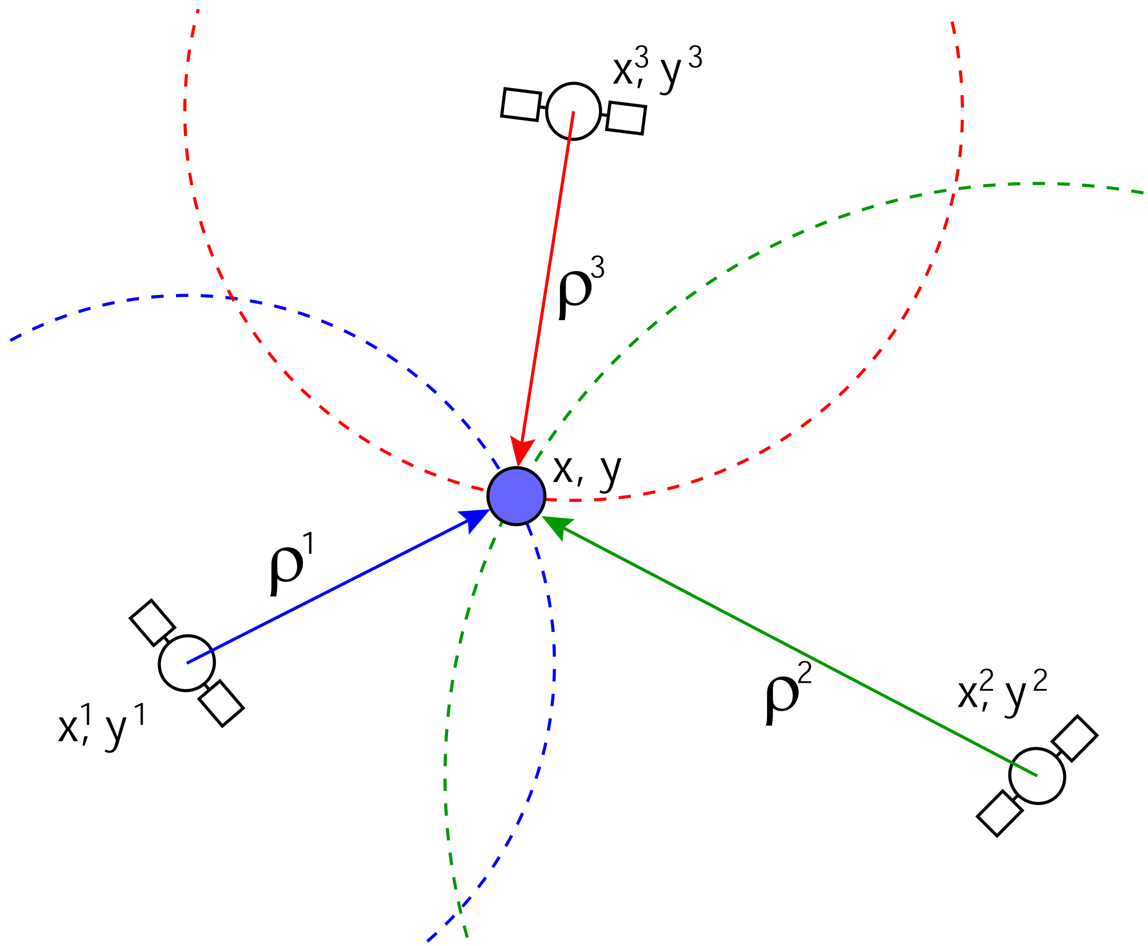


- At least 24 satellites in 12 hour orbits (26.5 kilometer)

User Segment

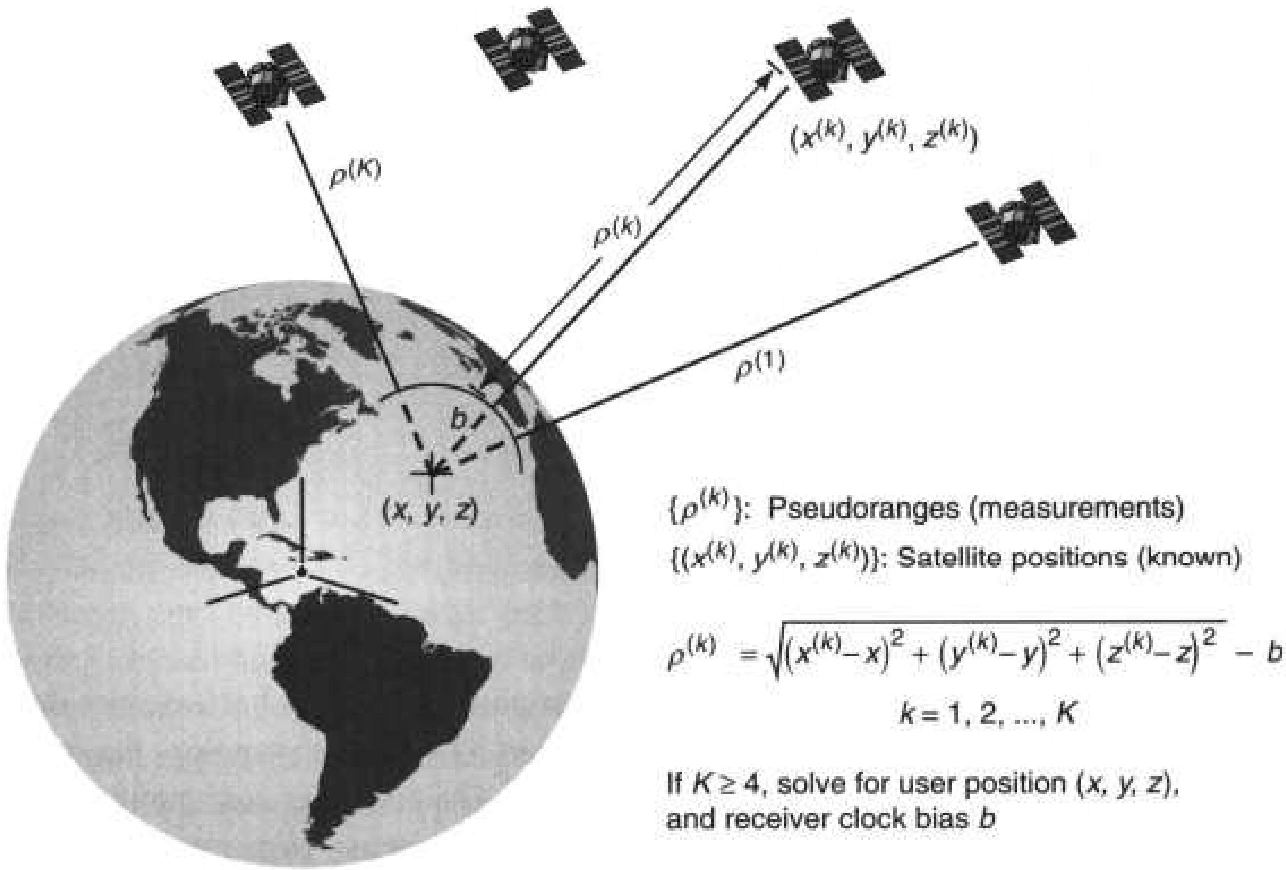
- Users are completely passive observers of GPS signals
- To get the most out of GPS, it helps to understand the governing physical principles

2D Position Solution



- User position is a function of satellite positions and distance from the user to each satellite (pseudoranges)

3D Position Solution



- Now spheres instead of circles

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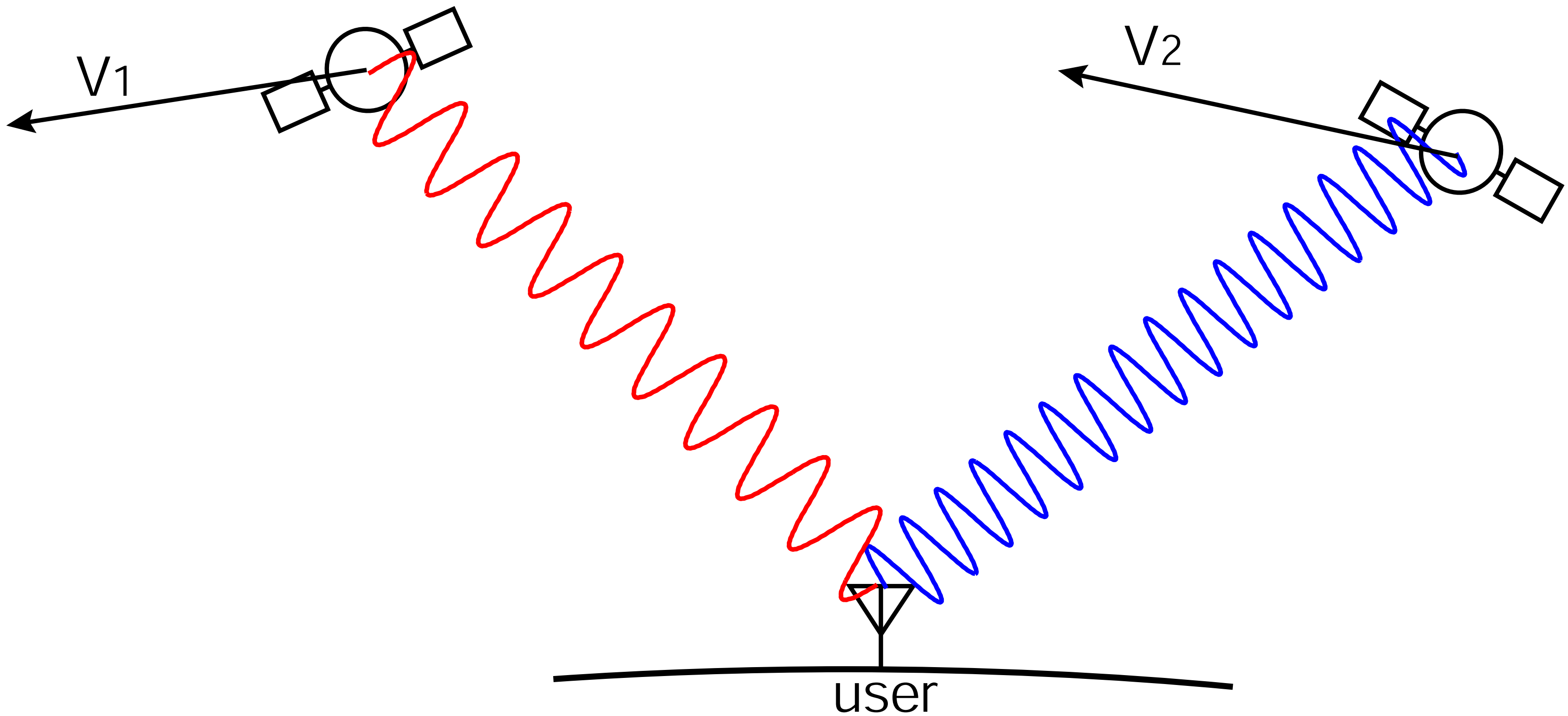
Position Solution: Data Transmission

- How are the satellite positions known?
  - They are uploaded by the control segment
  - Then transmitted by each satellite along the GPS carrier
- Currently two different GPS frequencies
  - L1 is the Coarse/Acquisition (C/A-Code) frequency (Civilian)
    - 1.575 Ghz
  - L2 is the Precision (P-code) frequency, (Military)
    - 1.227 Ghz
    - Encrypted

Data Transmission

- L1 transmits quite a lot of data
  - Timing information
  - Satellite orbit information
  - Almanac of other satellite locations
  - Ionosphere models
  - Uses CDMA data encoding
- Problem: The satellites are not stationary, they are in 12 hour orbits
  - They are moving fast enough to shift the apparent carrier frequency

Doppler Shift



- Phase Lock Loops track the doppler frequency shift
  - A byproduct of these loops is an accurate measurement of $\frac{d}{dt}\rho^k$
  - (The pseudorange rates provide a nice velocity measurement)

Position Solution: Pseudoranges

- How are the pseudoranges calculated?
  - The GPS data transmits the time the signal was sent
  - (Accomplished by looking at bit transitions of CDMA data)
  - The receiver records the time received
  - The pseudorange, $\rho^k \approx c\Delta t$, where c is the speed of light
- The distance from the user to the satellite then
  - $\rho^k \approx \sqrt{(x^k - x)^2 + (y^k - y)^2 + (z^k - z)^2}$
- This requires very precise timing information

Position Solution: Light Speed

- The pseudorange, $\rho^k \approx c\Delta t$
  - $c \approx 3 \times 10^8 \frac{m}{s} \Rightarrow 1\mu s = 300$ m of error
  - GPS needs extremely accurate timing information
  - 3 m of error $\Rightarrow$ know time within 10 ns !
  - This is a fundamental limitation of range based solutions
- The satellites have Cesium and Rubidium clocks with a drift of 10^{-13} seconds per day
  - They cost several $100k each
- Problem: users use crystal clocks which drift by $\frac{ms}{\text{day}}$

Position Solution: Clock Biases

- Problem: users use crystal clocks which drift by $\frac{ms}{\text{day}}$
- Solution: The clock bias is common to all of the ρ^k
  - Use an extra measurement to solve for the bias
  - $\rho^k = \sqrt{(x^k - x)^2 + (y^k - y)^2 + (z^k - z)^2} - b$
- This is why GPS needs 4 satellites for a position solution
  - Need to estimate x, y, z, b
- This is a well behaved Nonlinear Least Squares Problem

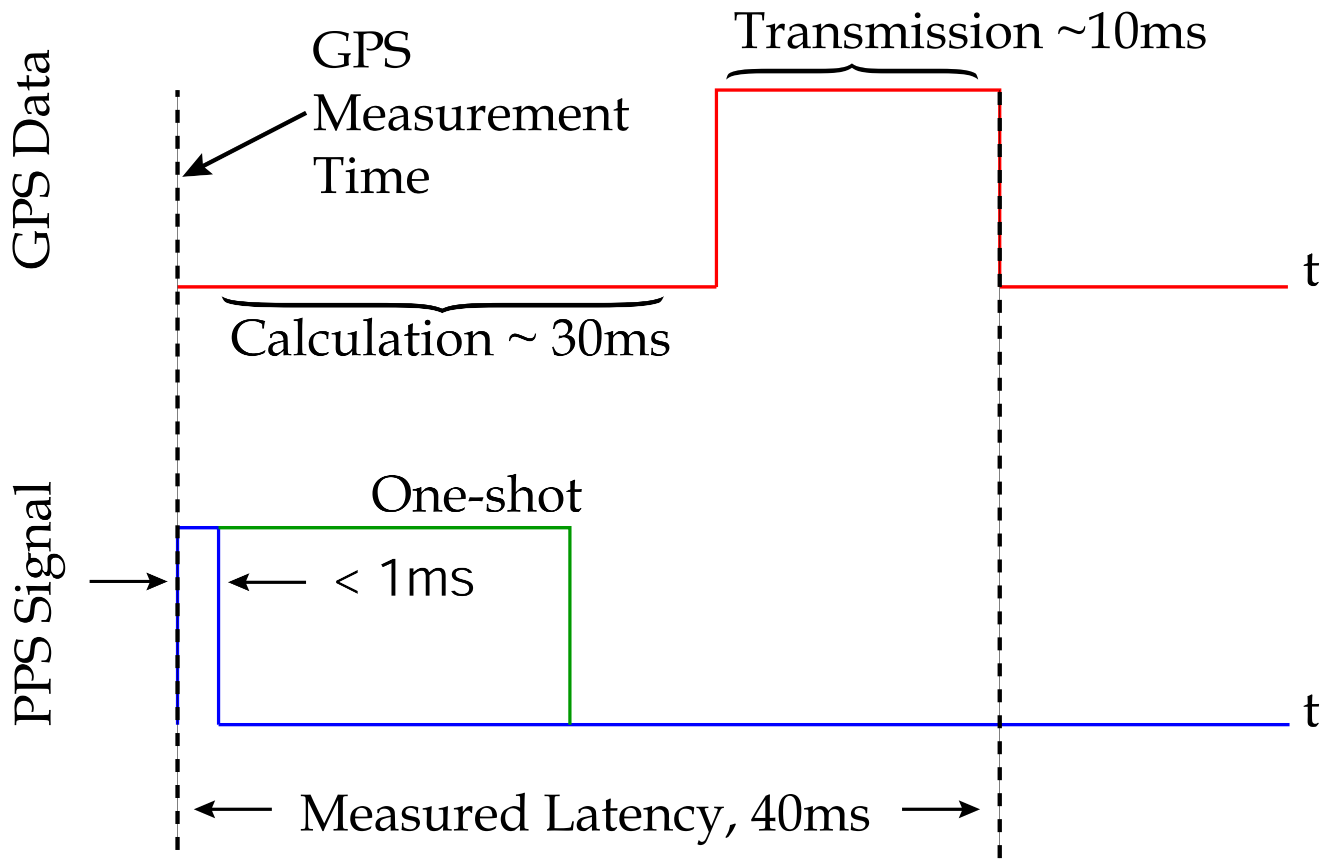
Position Solution: Algorithm

- $\rho^k = \sqrt{(x^k - x)^2 + (y^k - y)^2 + (z^k - z)^2} - b$
  - Minimum of 4 measurements needed
  - $x^k, \; y^k, \; z^k, \rho^k$ are all known
  - Wish to solve for $x, \; y, \; z, \; b$
- Let $x^k = [x^k, y^k, z^k]^T$, and $x_0 = [x, y, z]^T$
  - $\rho^k = \|x^k - x_0\| - b$
  - $\rho^k \cong \|x^k - x_0\| + \frac{(x^k - x_0)^T}{\|x^k - x_0\|}(\Delta x) - b$
  - $[x_0^T, b]^{T,j+1} = [x_0^T, b]^{T,j} - ([\frac{(x^k - x_0)^T}{\|x^k - x_0\|}, 1])^\dagger(\rho^k - \|x^k - x_0\| + b)$

Position Solution: Pulse Per Second

- GPS “knows” precise timing information
  - Good GPS receivers know timing within tens of ns
- However : The GPS measurement is not output instantaneously! It must,
  - Evaluate the orbit information, etc
  - Solve the NLLS problem
  - Assemble and transmit the data log
- Good receivers output PPS information to compensate
  - This makes the latency of the system measureable

PPS



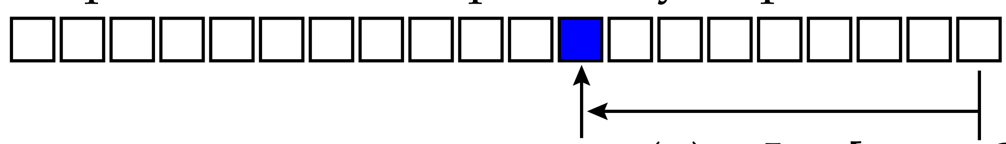
- Analog PPS signal enables latency calculation
  - May need to extend pulse to greater than sample period

Accounting for Latency

Step 0: Record real-time data in a buffer

Step 1: Record new GPS measurement

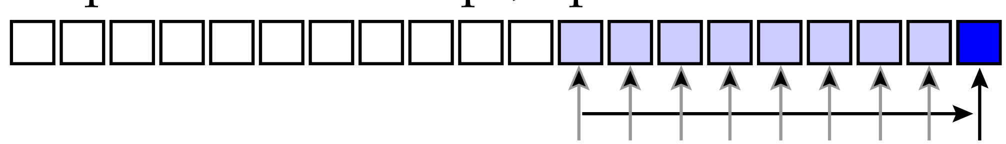
Step 2: Corrector step, latency steps behind



$$x_{k-\ell}(+) = L_{k-\ell}[y_{k-\ell} - C_{k-\ell}x_{k-\ell}(-)]$$

$$P_{k-\ell}(+) = [P^{-1}_{k-\ell}(-) + C^T_{k-\ell}R_{k-\ell}C_{k-\ell}]^{-1}$$

Step 3: Predictor steps, up to current time



$$x_{k-j+1}(-) = A_{k-j}x_{k-j}(+) + B_{k-j}u_{k-j}$$

$$P_{k-j+1}(-) = A_{k-j}P_{k-j}(+)A^T_{k-j} + Q_{k-j}$$

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Principle Error Source

- The sky is totally or partially occluded
  - Need a minimum of 4 satellites for a position solution
  - Most of the time we have more than that
  - Good GPS receivers know when their measurements are poor
- We consider GPS a signal of opportunity
  - When GPS is available, take advantage of it
    - Estimate parameters
    - Diagnose faults
  - When GPS is not available
    - Rely on (well tuned) internal observers

Principle Error Sources: SA

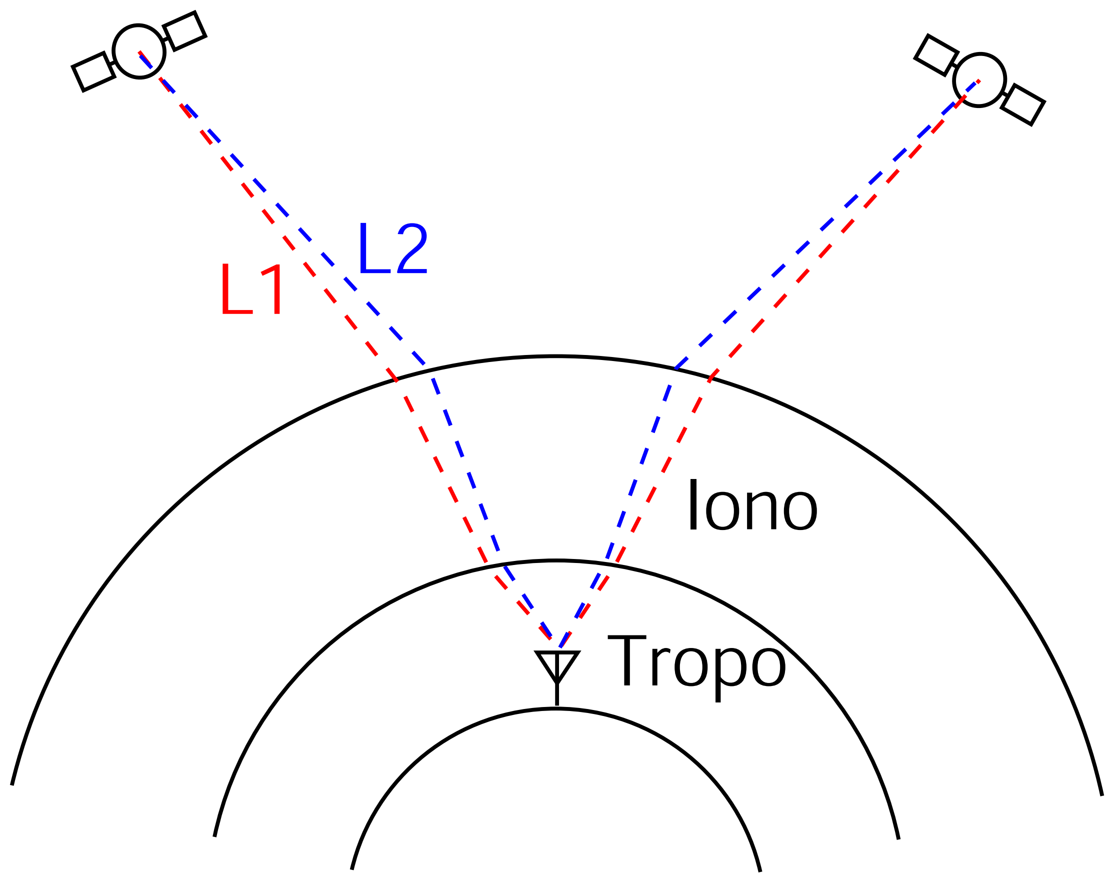
Selective Availability (SA)

- Deliberate white noise added to the clock information before transmission from the satellites
- Stand alone GPS was good to about 100 m
- Clinton turned SA off in May 2000
- Stand alone GPS is now as good as 1.8m CEP

CEP : Circular Error Probability

- 50% of measurements fall inside circle
- 1 $\sigma \Rightarrow 64\%$, so CEP is more aggressive than 1σ

Principle Error Sources: Iono

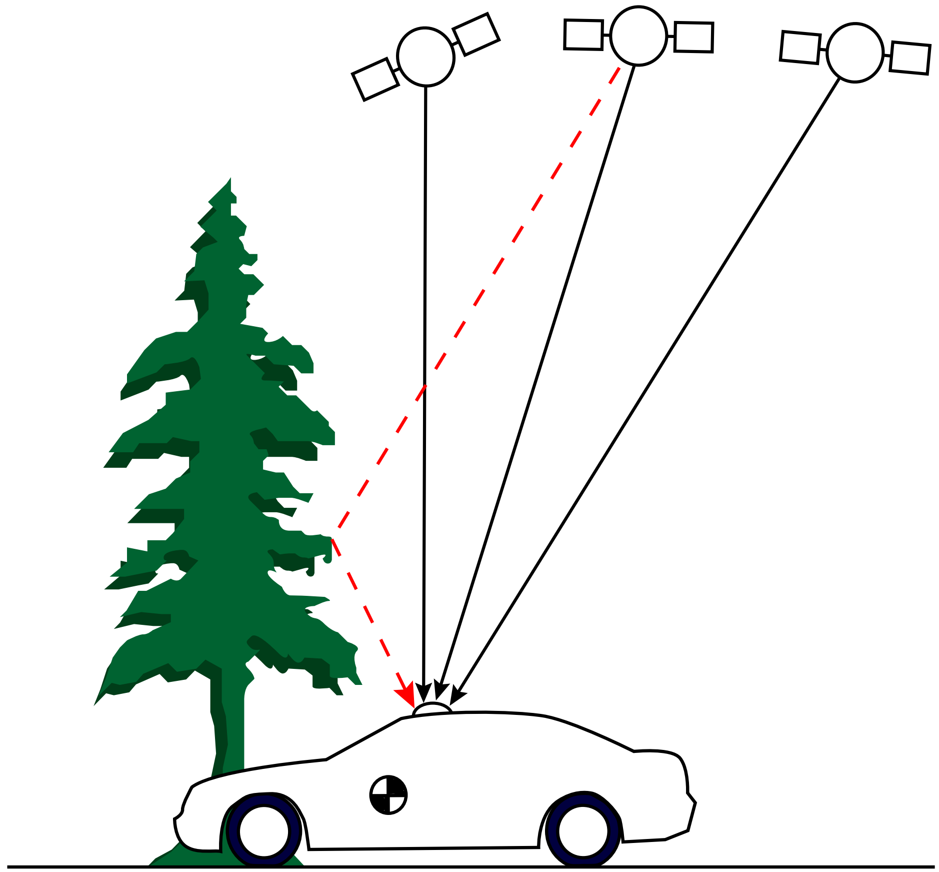


- Ionosphere and Troposphere lengthen GPS carrier path
  - About 50% of this error may be compensated with a model

Principle Error Sources: Iono

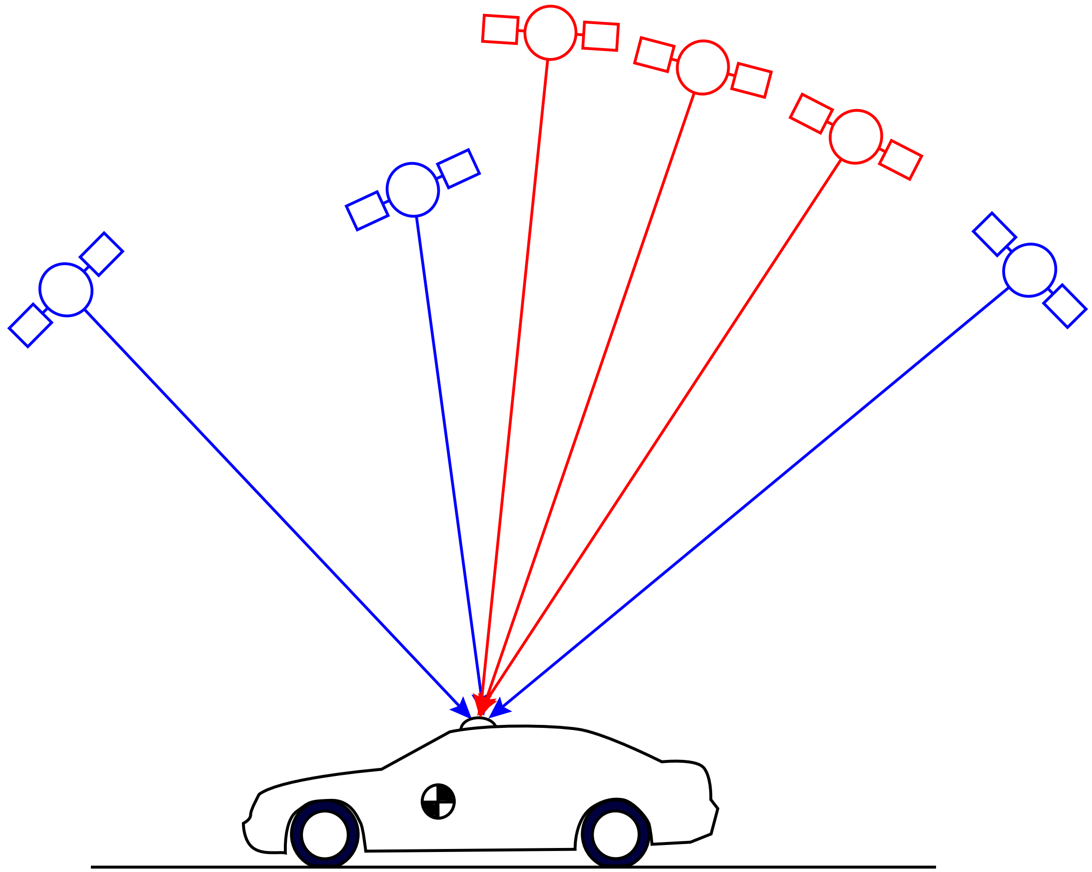
- L1 and L2 are different frequencies
  - They are each effected by the ionosphere and troposphere differently
  - Just like red light and blue light travelling though a prism
- If measuring both L1 and L2, it is possible to estimate the atmospheric disturbance
  - The military uses this to their significant advantage
  - This is somewhat possible even for civilian users
  - L2 tracking is an option on most high end receivers
    - Requires double the hardware (and costs more than double)

Principle Error Sources: Multipath



- Reflected GPS signals have longer pseudoranges
  - Antenna placement and clever algorithms are all we can do

Principle Error Sources: Satellite Geometry



- Better measurements from diverse satellite geometry
  - Vertical measurements ~50% less accurate than horizontal

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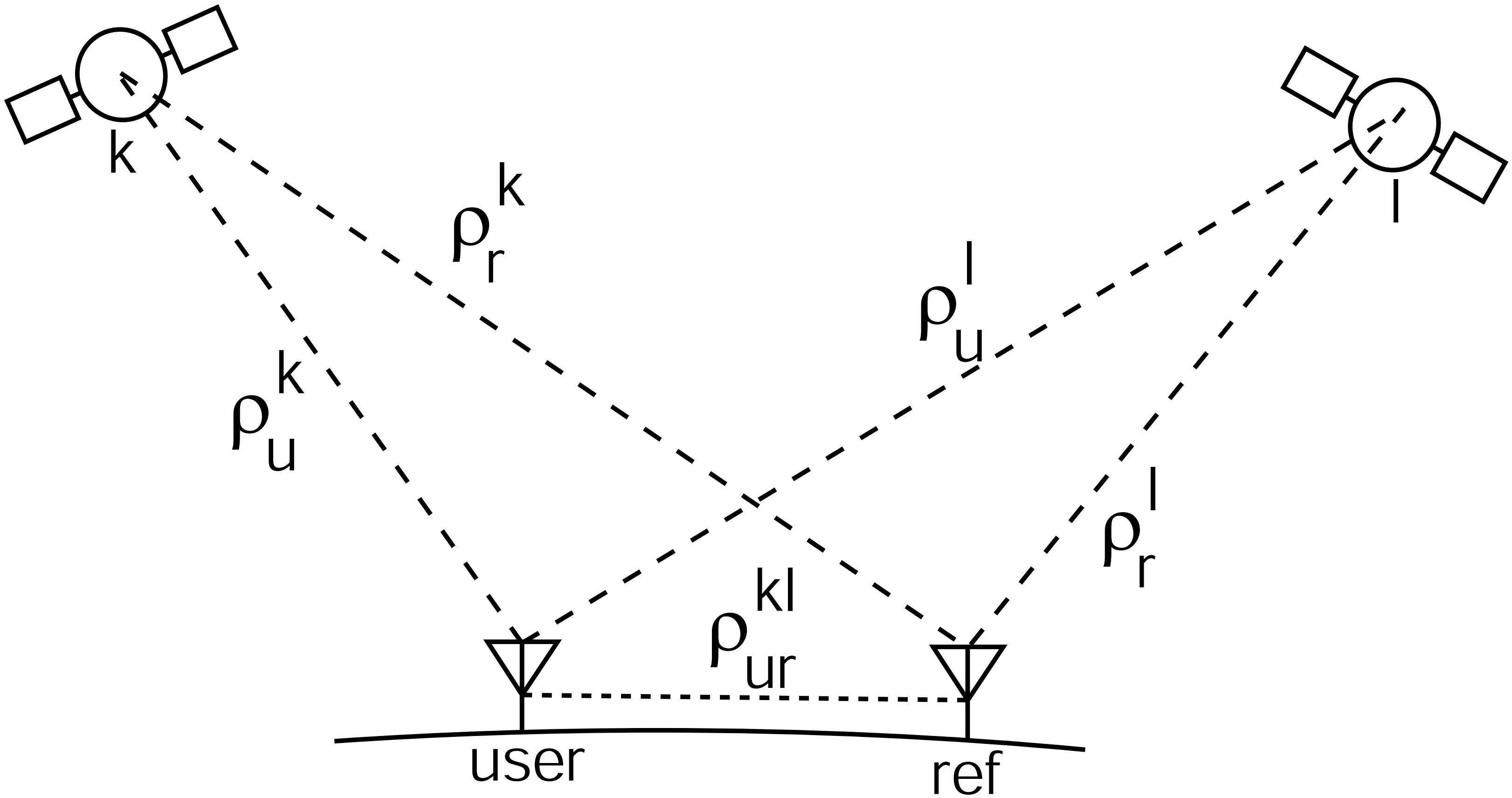
DGPS

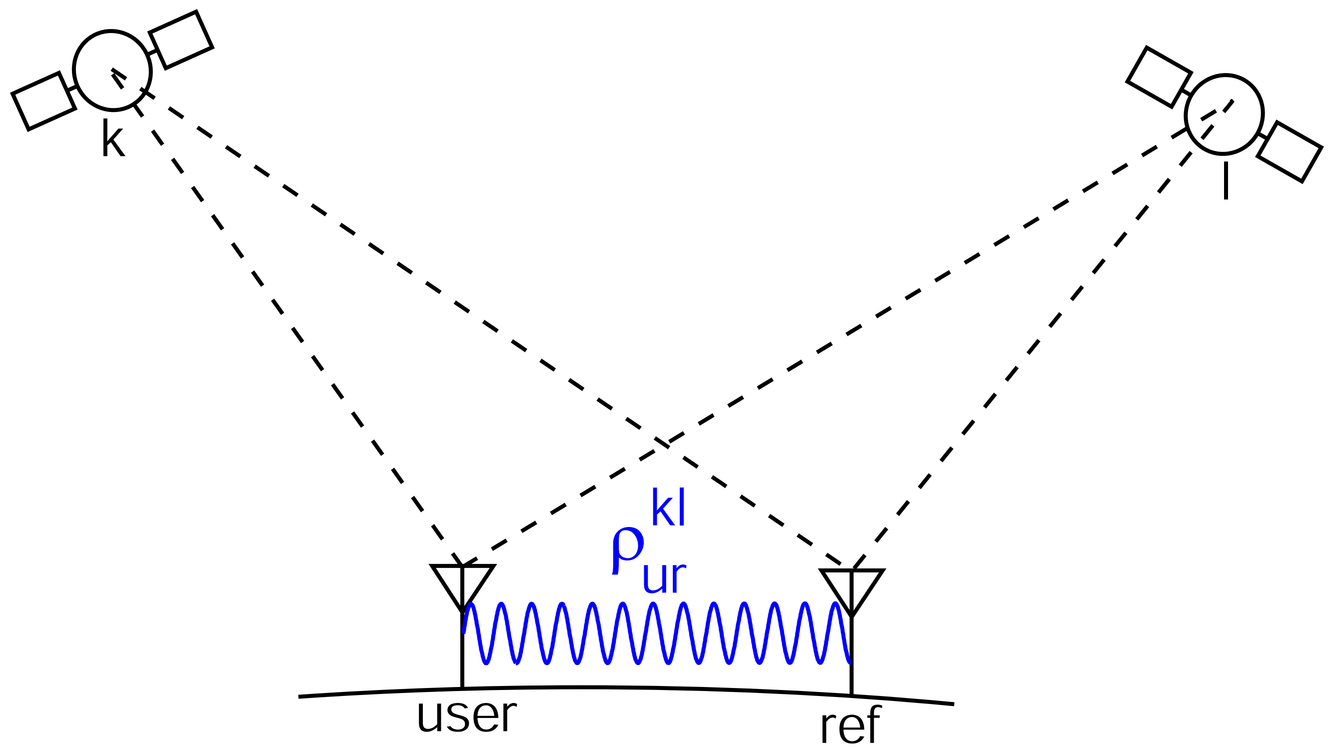


- Cancels out common mode errors (primarily Ionosphere)
  - Either "Code Phase" or "Carrier Phase"

DGPS: Code Phase

- SBAS: Space Based Augmentation System
  - Requires a special SBAS chip set
  - Currently good to about 1.2 m with L1 alone
  - good to about 0.8 m with L1 and L2
- Code phase, L1, C/A
  - Requires a data link between rover and base station
  - Good to about 0.5 m if done well

DGPS: Carrier Phase



$$\rho_{ur}^{kl} = 0.19\ N + f$$

- System tracks number of integer, N, plus fraction,f, wavelengths between user and reference
  - PLLs can resolve wavelengths to 1% $\Rightarrow$ 2mm accuracy

DGPS Carrier Phase

- Carrier phase, Real Time Kinematic, RTK, RT
  - Requires a data link between rover and base station
  - Good to about 20 cm with L1 (floating point problem)
  - Good to 2 cm with L1 and L2 (integer problem)
- Takes advantage of the carrier tracking loops to track the number of wavelengths between base station and rover
  - This is inherently a higher signal to noise ratio problem
  - It is also usually *much* more expensive

DGPS: Data Link

- DGPS requires a data link and a separate receiver
- Do it yourself, “Wireless Wire”
  - RS232 modem
  - Packet radio system
  - Cell phones (DaimlerChrysler Palo Alto)
  - Can support carrier phase
- Services
  - Omnistar, SBAS, etc
  - Usually only code phase

GPS Velocity

- GPS velocity is differential GPS in *time*
  - The principle errors due to the atmosphere are slowly varying
  - Therefore their derivative is small
  - GPS velocity in practice is accurate to about 2 $\frac{cm}{s}$
- There are two different kinds of GPS velocity
  - Single difference of position measurements
    - **Has an inherent *additional* latency of $\frac{Ts}{2}$**
  - Doppler
    - **Almost free via the carrier tracking loops**
    - **Instantaneous, no additional latency**

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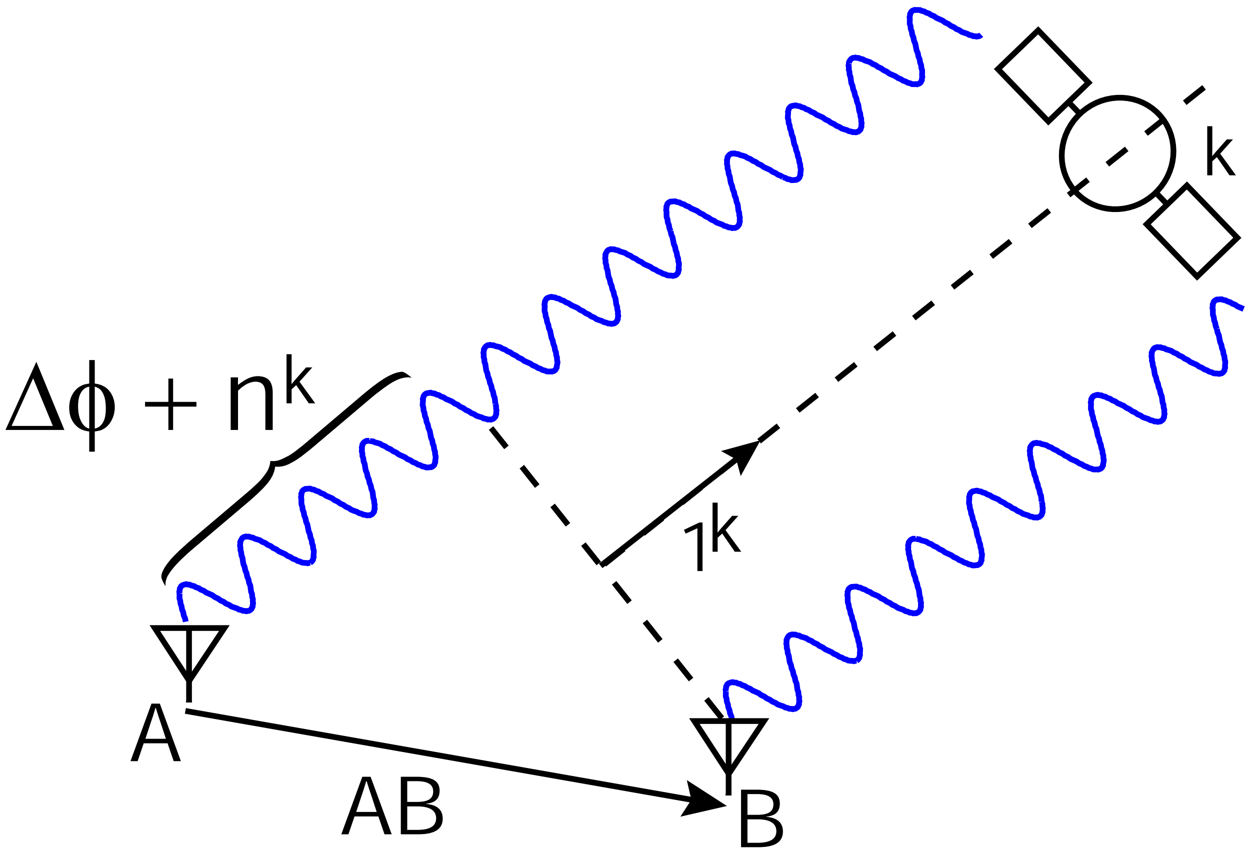
GPS Attitude



- The phase difference, $\Delta\phi^k + n^k$, may be written as a linear function of the known LOS (1^k) to the satellite
  - $\Delta\phi + n = 1^{kT}AB$

GPS Attitude

- Attitude is an integer filtering problem (Hard)
  - $\min_{N} [1^T AB - H^\dagger(\Delta\Phi + N)]$
- Accuracy depends upon the antenna separation
  - 1 m $\Rightarrow 0.15°,\ 1\ \sigma$
  - 0.5 m $\Rightarrow 0.5°,\ 1\ \sigma$
- May use 2, 3 or 4 antennas
  - 2 antennas define a line in space
  - 3 antennas define a plane
  - 4 antennas define a robust plane

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Summary

GPS Data sheets

Summary

- GPS needs at least 4 satellites for a valid solution
- PPS and latency are critical for dynamic applications
- GPS velocity: doppler and differenced
- Atmosphere and multipath are the principle error sources
- L2 aids performance (and costs more)
- DGPS provides significantly increased performance, needs a data link and a base station

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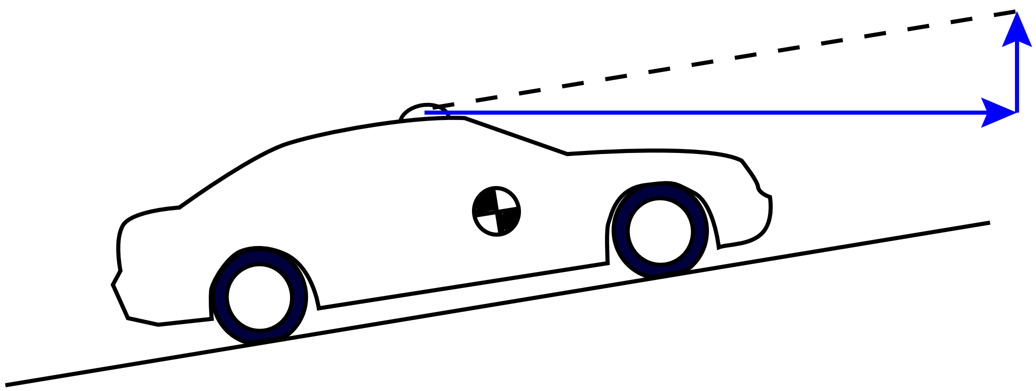
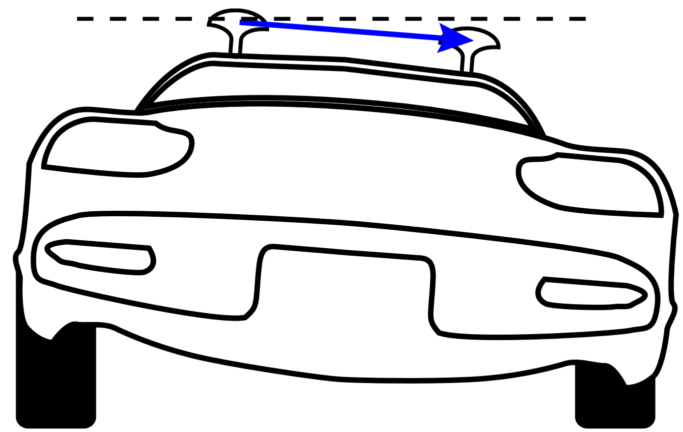
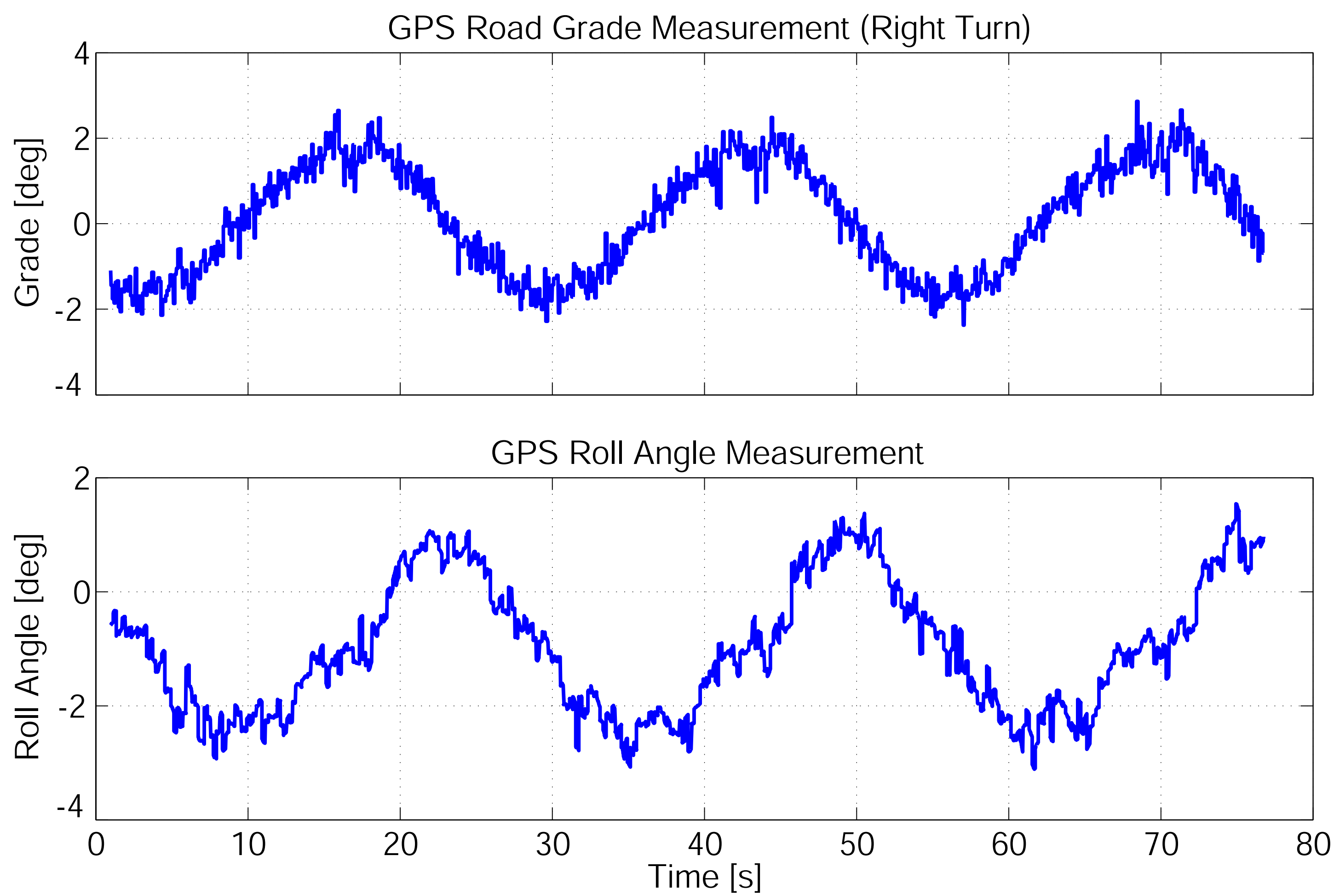
GPS Roll and Road Grade





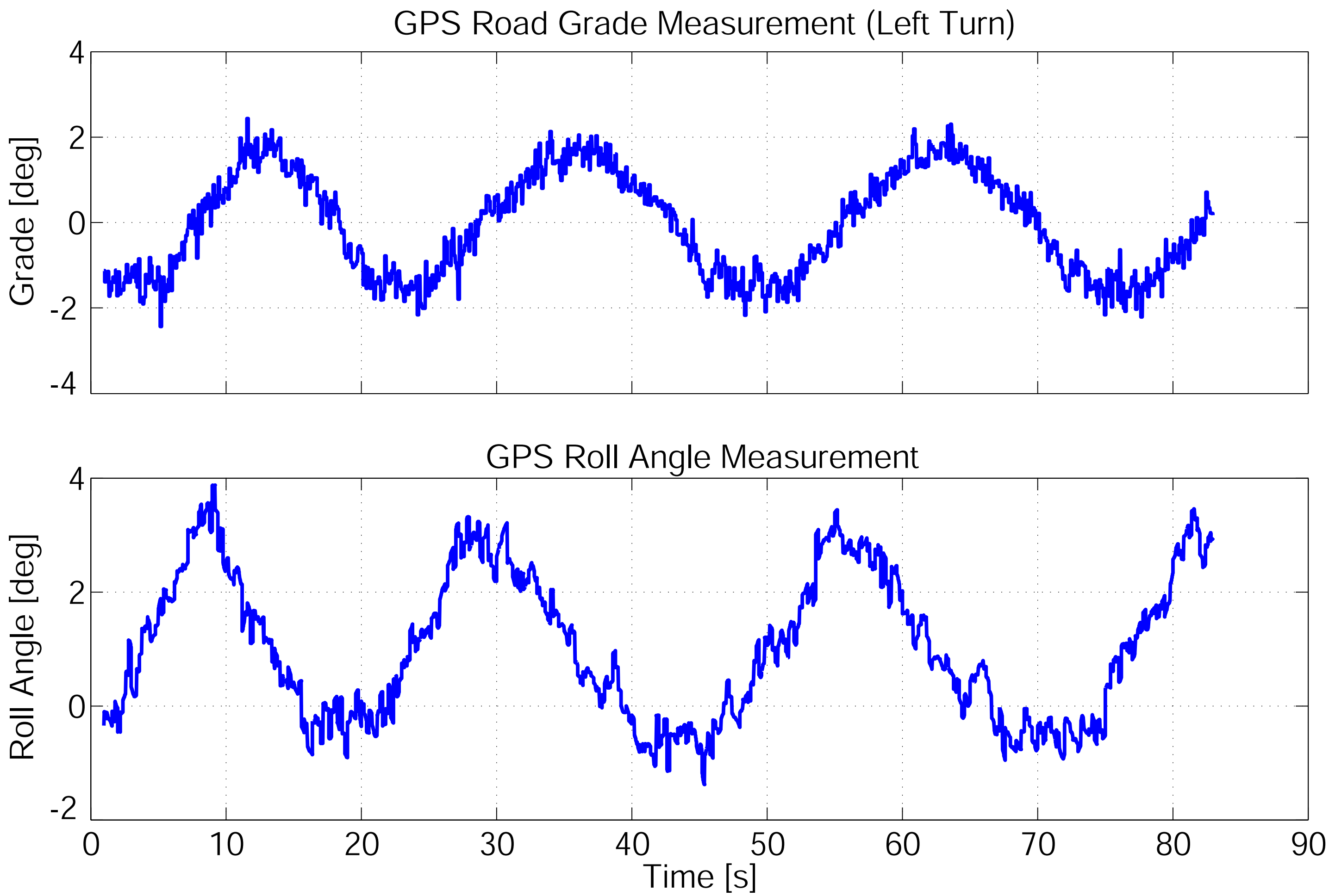
- 1 Antenna system measures velocities
- 2 Antenna system measures roll and heading

GPS Roll and Road Grade



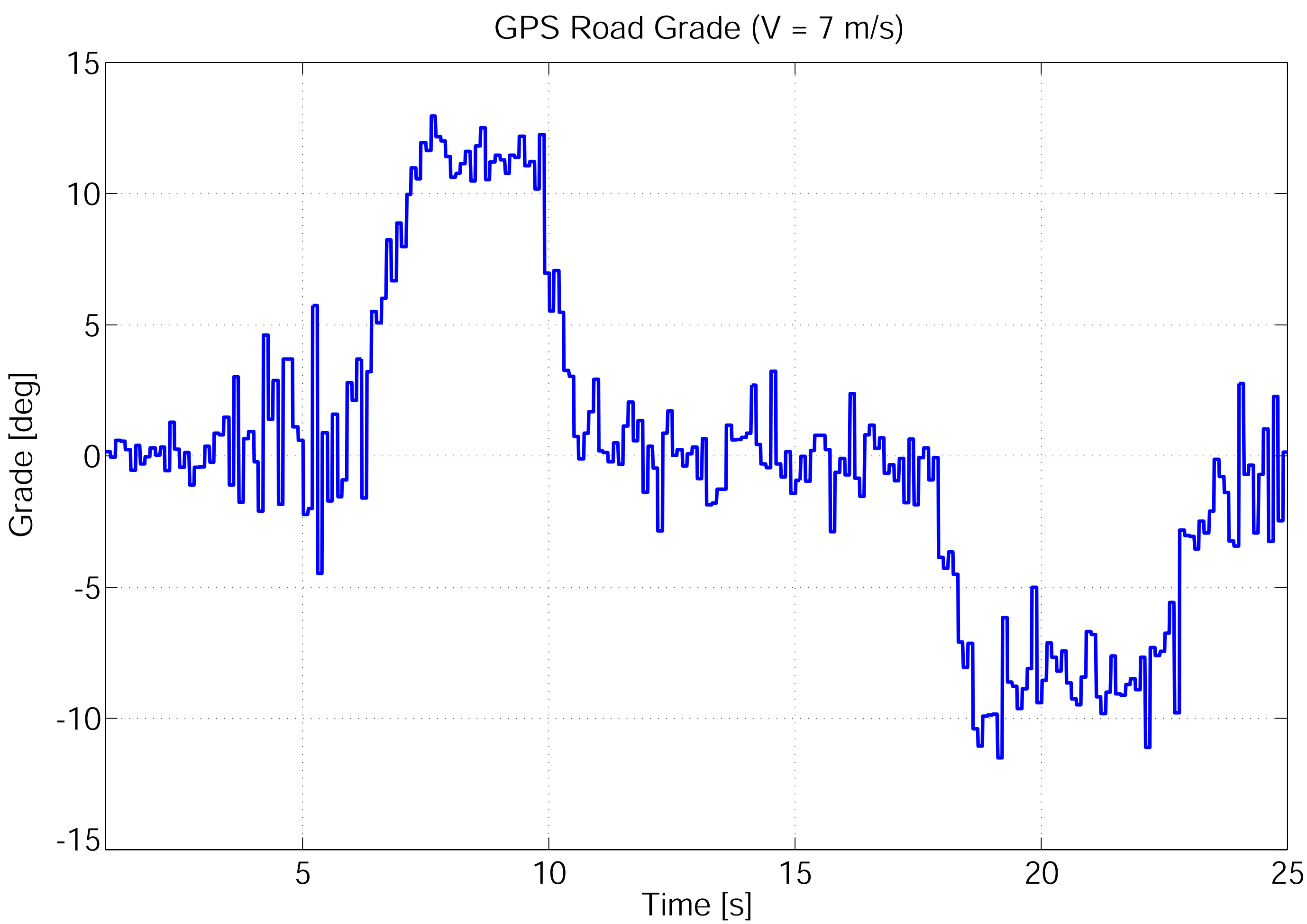
- Continuous 30km/h circle spanning Malmsheim track
- Road crown is clearly measurable as roll and grade

GPS Roll and Road Grade



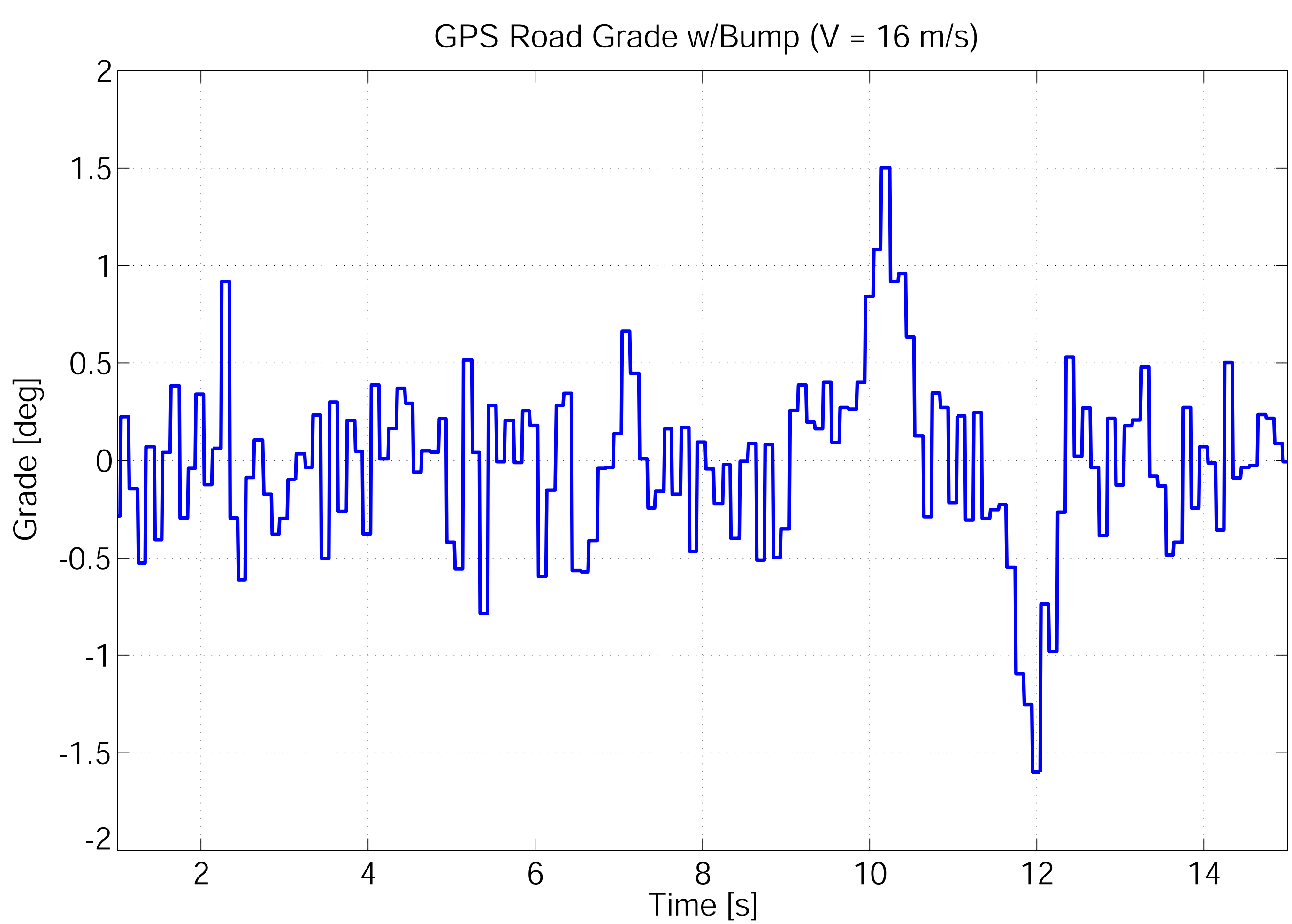
- Continuous 30km/h circle going the opposite direction
- 1° bias in roll angle probably due to ABC

Known Road Grade ?



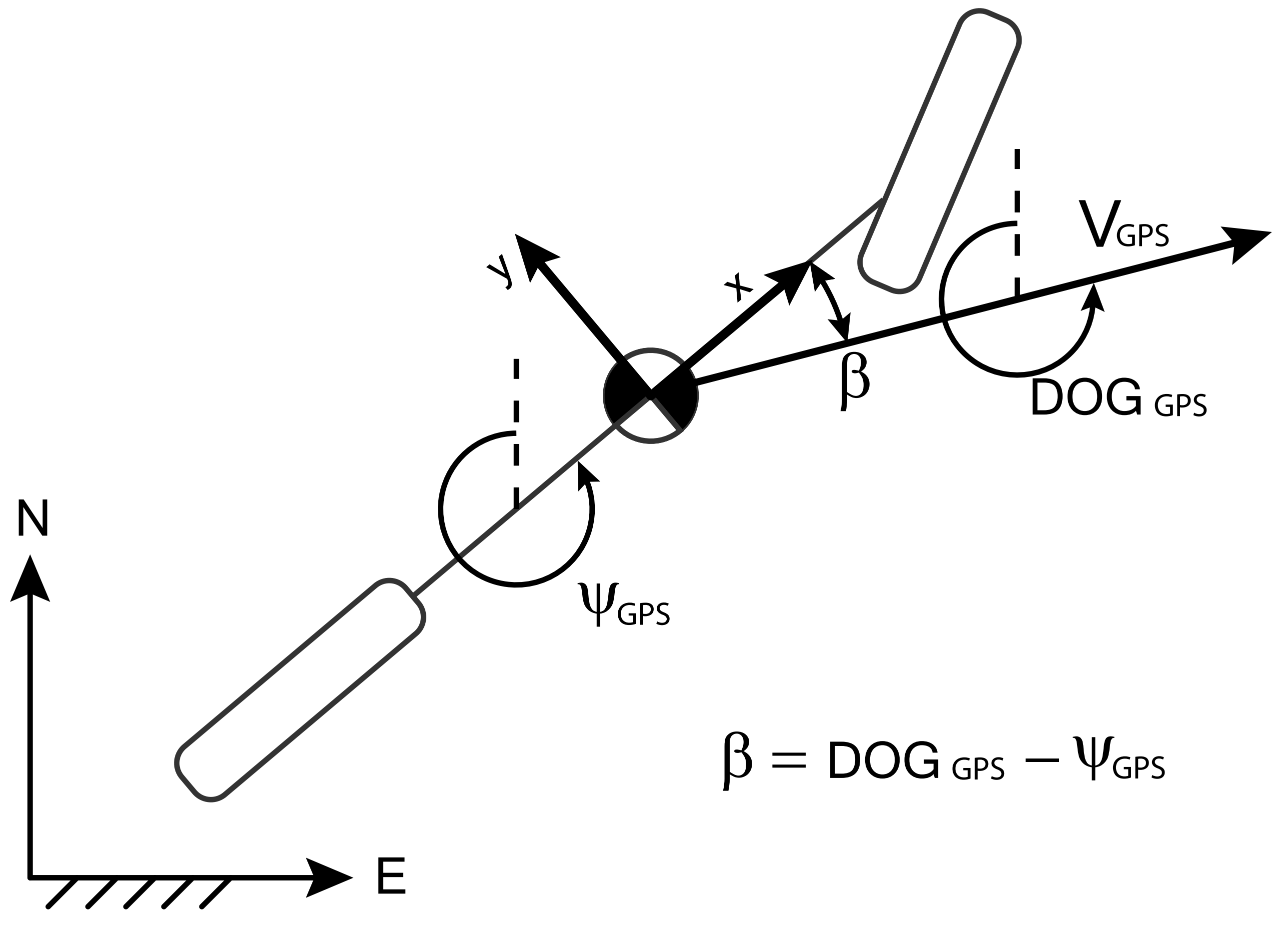
- Known road grade at Unter-Turkheim test track
- $\tan(11.5^\circ) = 0.2$, $\tan(8.5^\circ) = 0.15$

Road Grade with Bump



- Velocity measured at the *antenna* location
- Faster dynamics capturable with INS

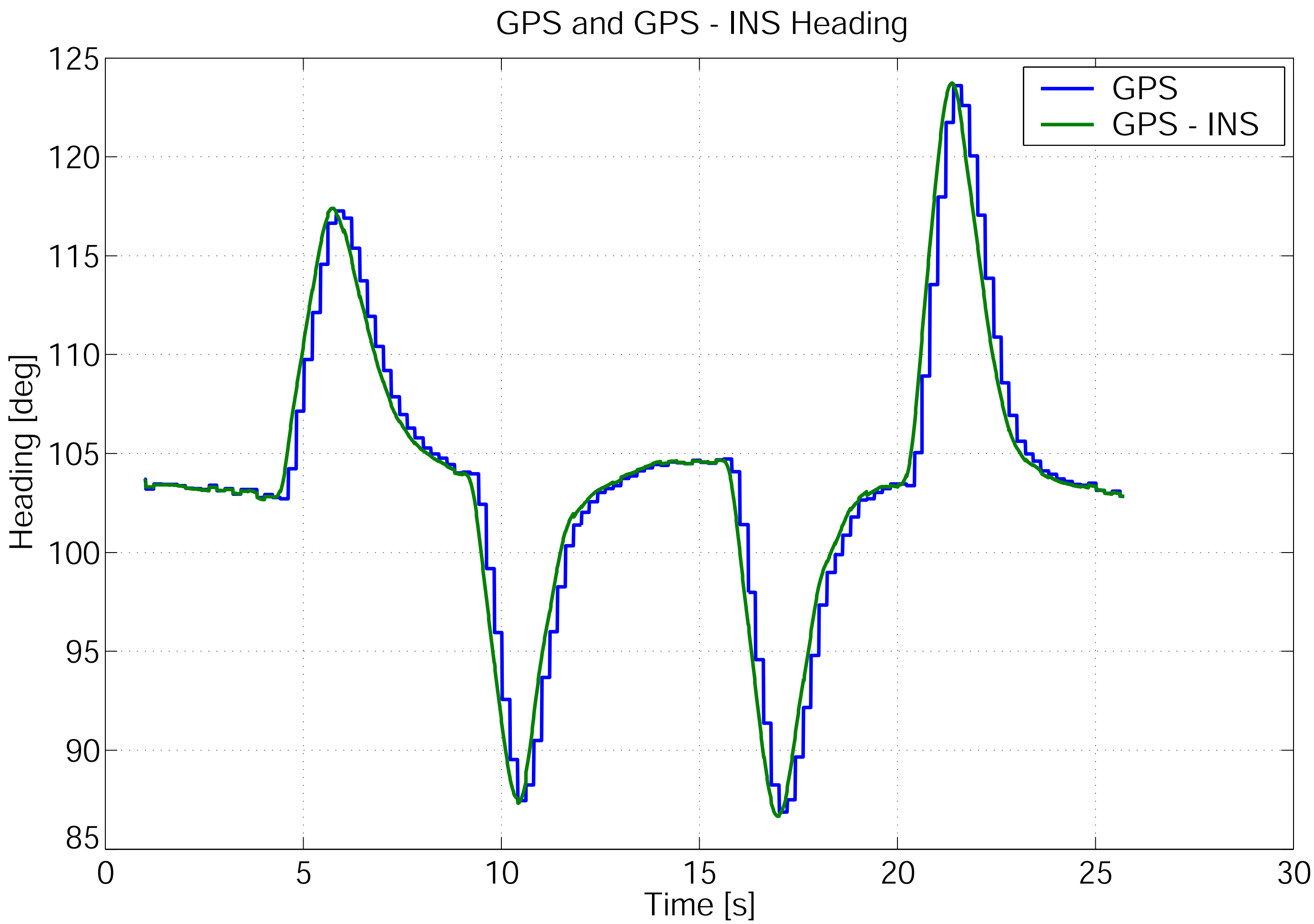
GPS Heading and Sideslip



$$\beta = DOG_{GPS} - \psi_{GPS}$$

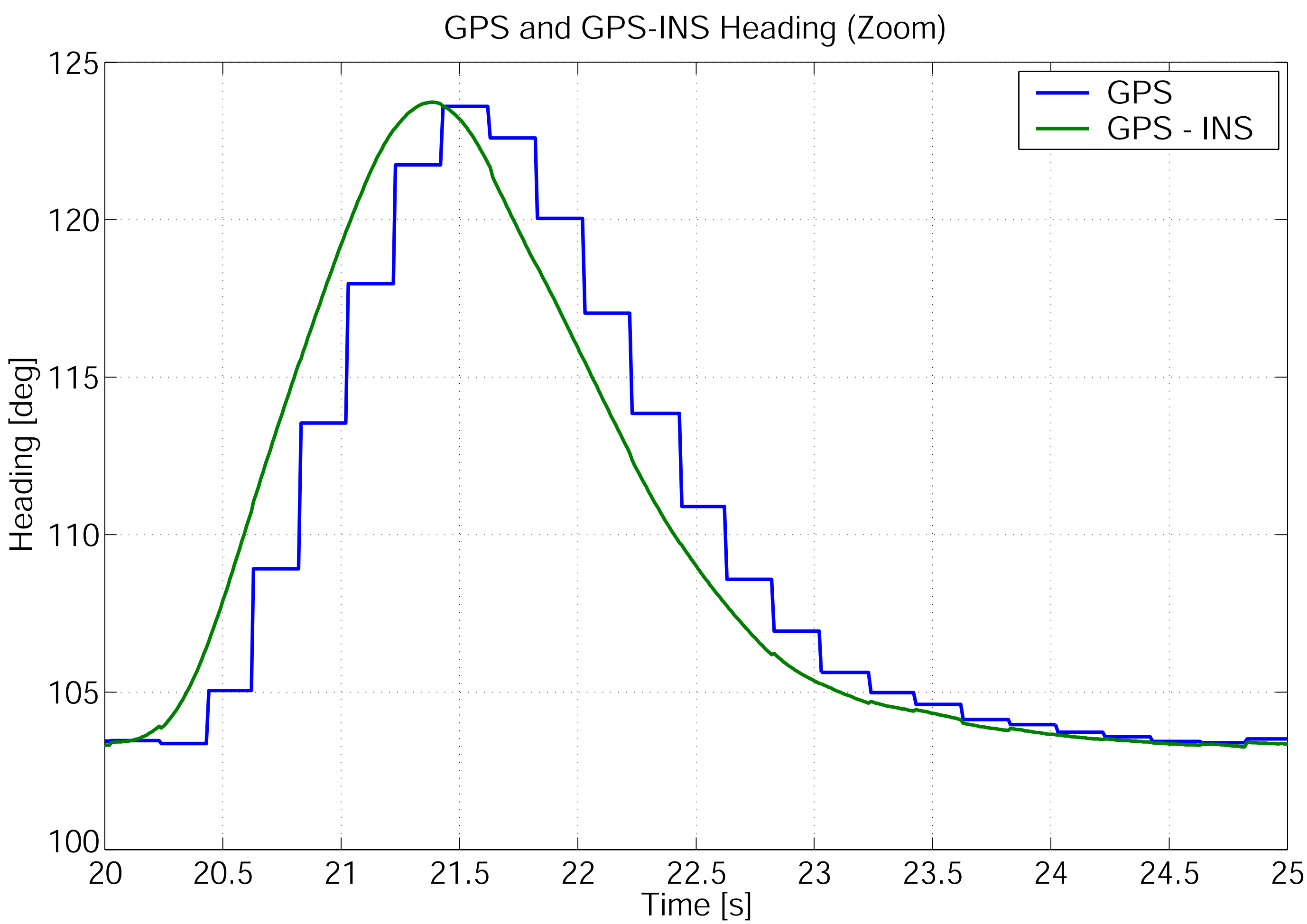
- GPS measures Heading, Roll, Speed and DOG
- INS measures yaw rate, roll rate, ax, ay

GPS Heading Estimation



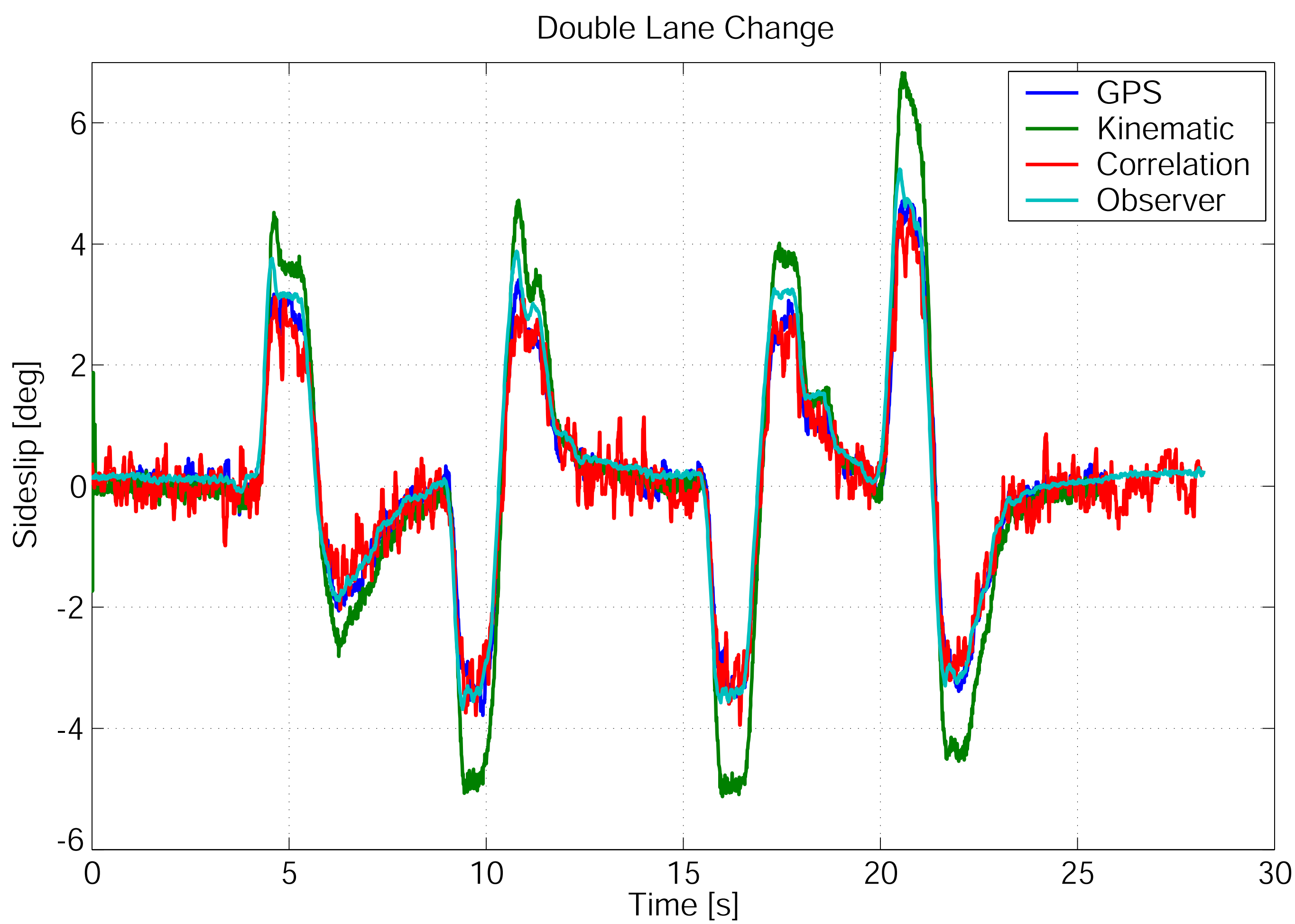
- Double lane change maneuver
- Heading measurement has a high signal to noise ratio !

GPS Heading Estimation



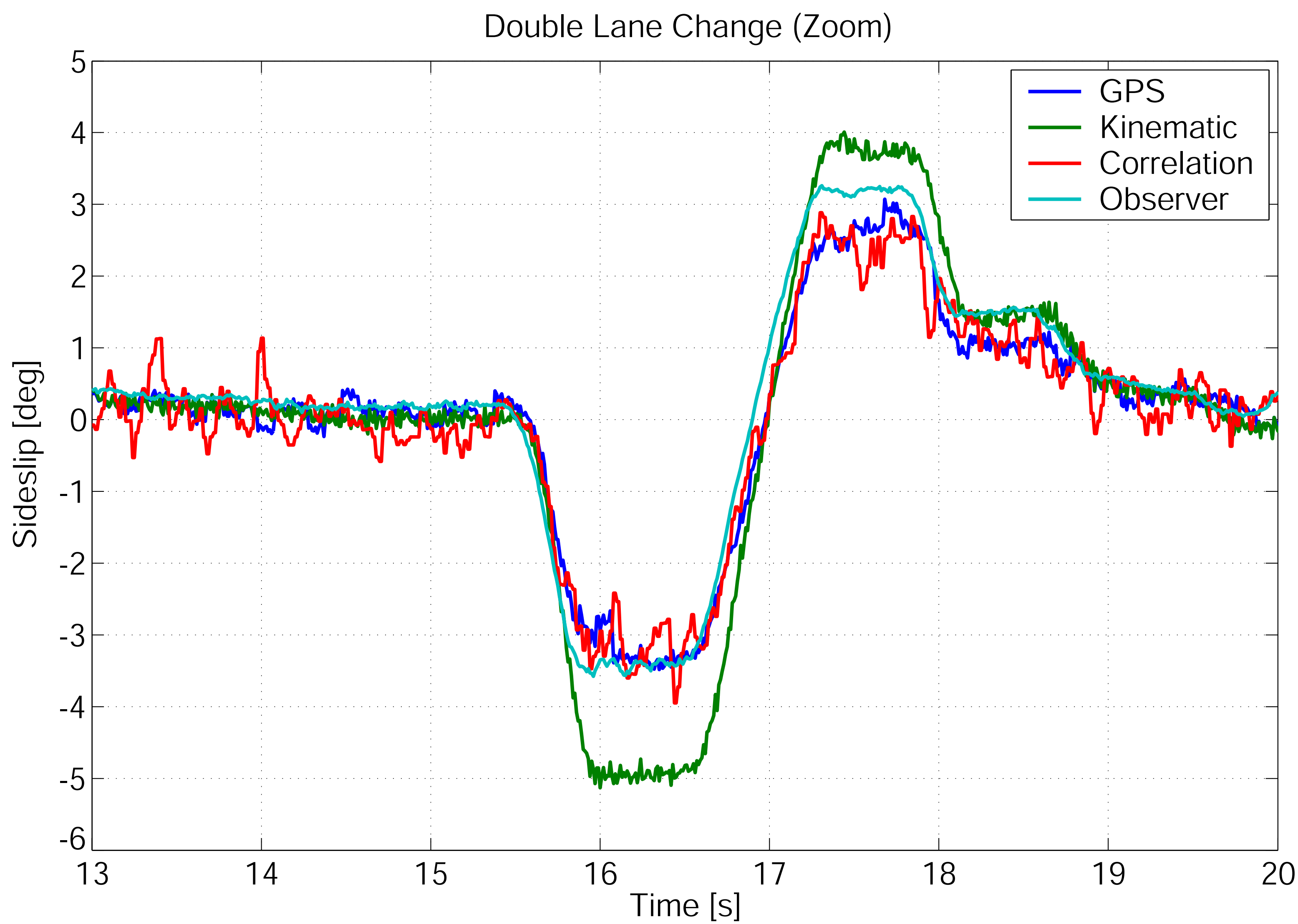
- Can see slight corrections by GPS
- What causes the delay?

Double Lane Change



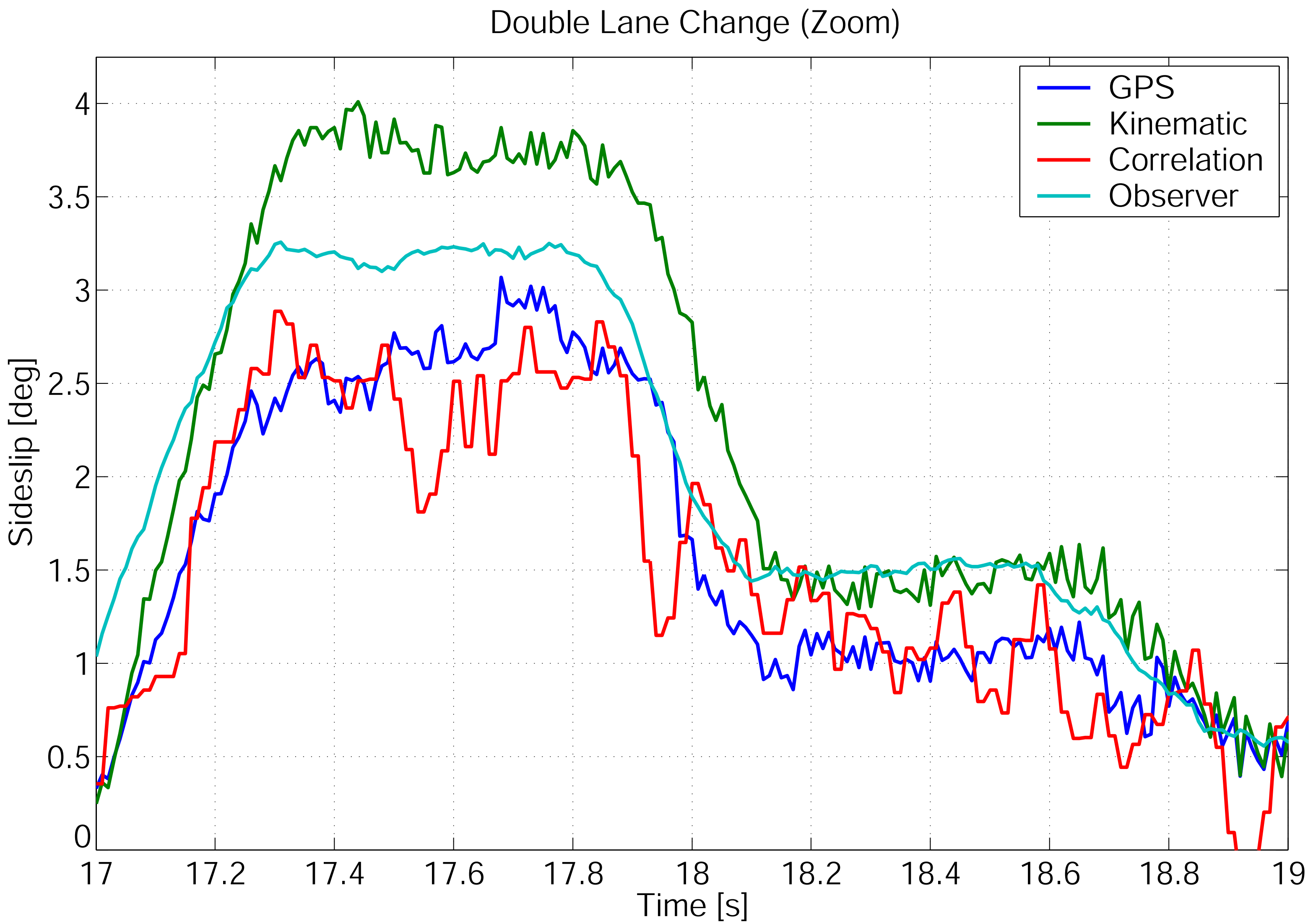
- 50 km/h double lane change at Malmsheim
- Kinematic model does not hold!

Double Lane Change Zoom



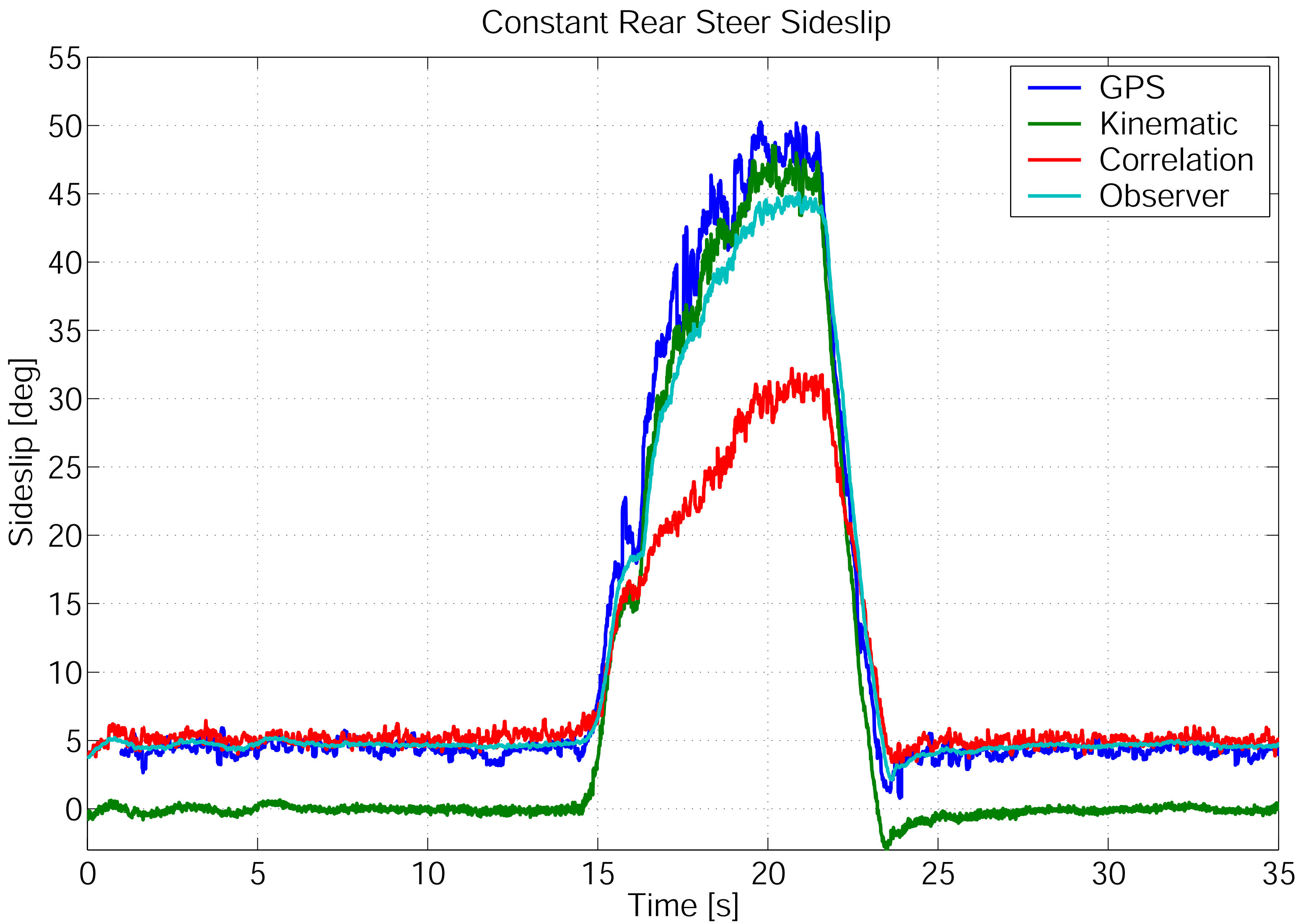
- Signals are slightly offset, geometry assumptions ?

Double Lane Change Zoom^2



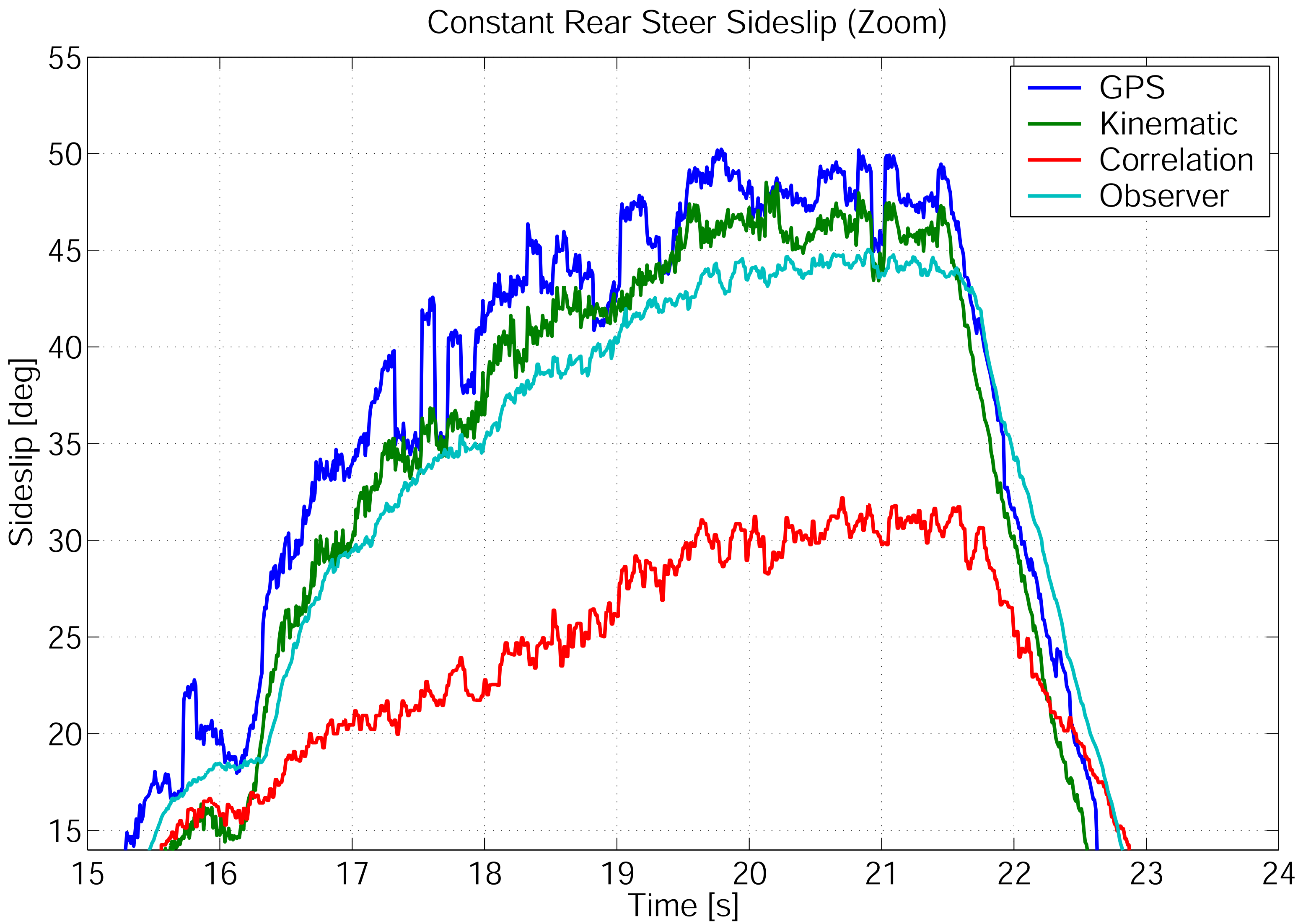
- GPS sideslip measurement looks precise to about $0.2°$

Constant Rear Steer



- Clearly Not Kinematic

Constant Rear Steer Zoom



- GPS DOG measurement error is speed dependent

Final Slide

Questions?