Identifying Tire Pressure Variation by Nonlinear Estimation of Longitudinal Stiffness and Effective Radius

Christopher R. Carlson
Stanford University

J. Christian Gerdes
Stanford University

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Dynamic Design Laboratory

Goal

Robust identification of tire properties using stock vehicle sensors

- ☐ Nearly all new cars have ABS
- ☐ GPS already prevalent and soon standard
- ☐ Tire longitudinal stiffness depends on many factors
  - Inflation pressure
  - Tread depth
  - Normal load

Applications

□ Onboard diagnostics

- Pressure sensor
- Tread wear

□ Peak traction force

- Stability control
- Driver warning systems

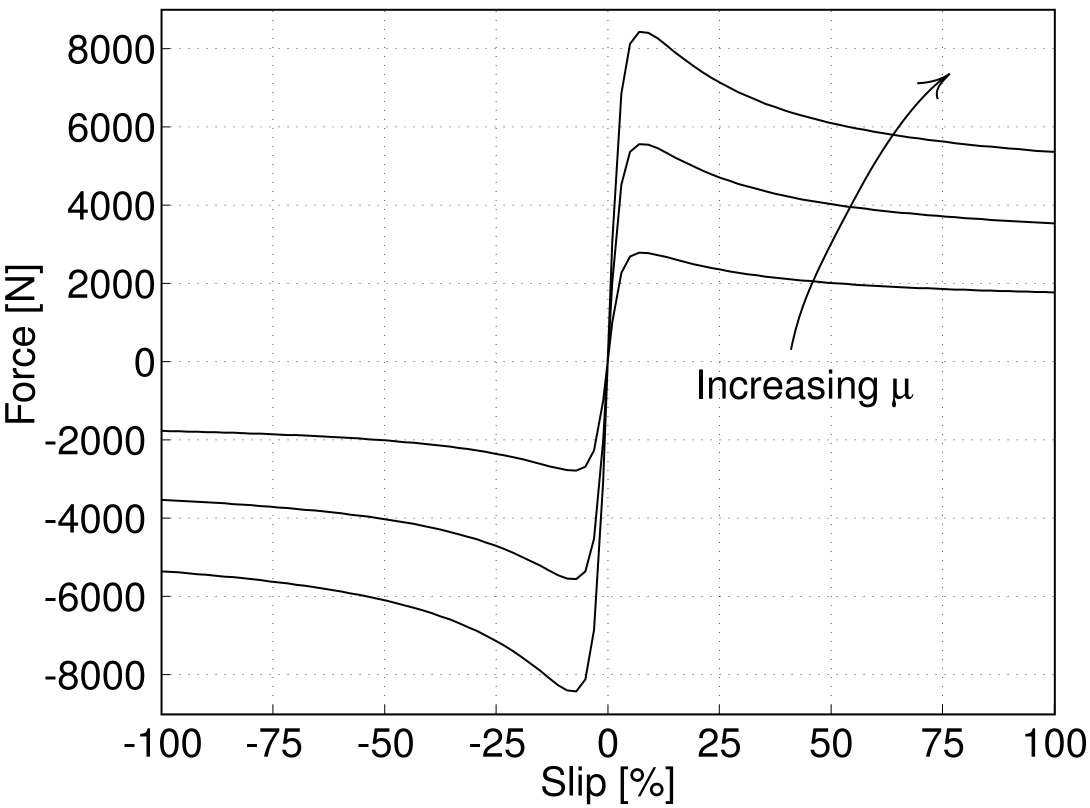
□ Tire modeling for simulation parameters

Outline

- □ Force Slip Model
- □ Available Sensors
- □ New Estimator Structure
- □ Results
- □ Conclusions
  - Much more consistent estimator
  - Longitudinal stiffness inversely proportional to inflation pressure

Force-Slip Model

Force Vs. Slip

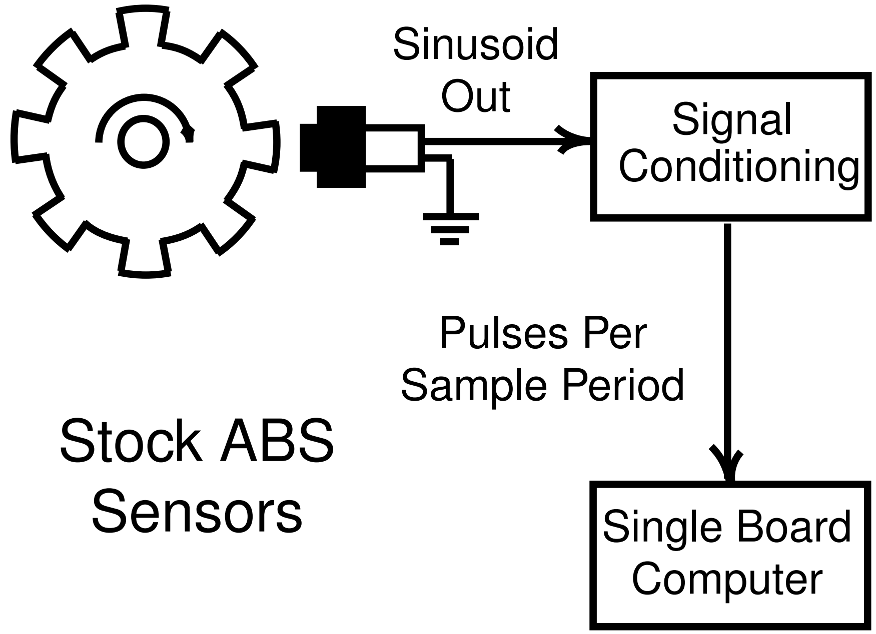


Force Vs. slip approximately linear for low values of slip

$$F = C_x \left(\frac{V - R\omega}{V} \right)$$

Available Sensors

Novatel GPS Receiver



Stock ABS Sensors

Measurements

$$\hat{V}^k = R_r \left(\frac{\hat{\theta}_r^{k+1} - \hat{\theta}_r^{k-1}}{2T} \right)$$

Velocity approximation

$$\hat{a}^k = R_r \left(\frac{\hat{\theta}_r^{k+2} - 2\hat{\theta}_r^{k} + \hat{\theta}_r^{k-2}}{4T^2} \right)$$

Acceleration approximation

$$\hat{w}_d^k = \left(\frac{\hat{\theta}_f^{k+1} - \hat{\theta}_f^{k-1}}{2T} \right)$$

Angular Velocity approximation

Previous Work

$$F = C_x \left(\frac{V - R\omega}{V} \right)$$

$$\Rightarrow$$

$$\hat{a} = \begin{bmatrix} -\frac{1}{m} & \frac{\hat{\omega}_f}{m\hat{V}} \end{bmatrix} \begin{bmatrix} C_x \\ R_f C_x \end{bmatrix}$$

Least squares parameter estimation

Had large variance of C_x estimates for similar test conditions

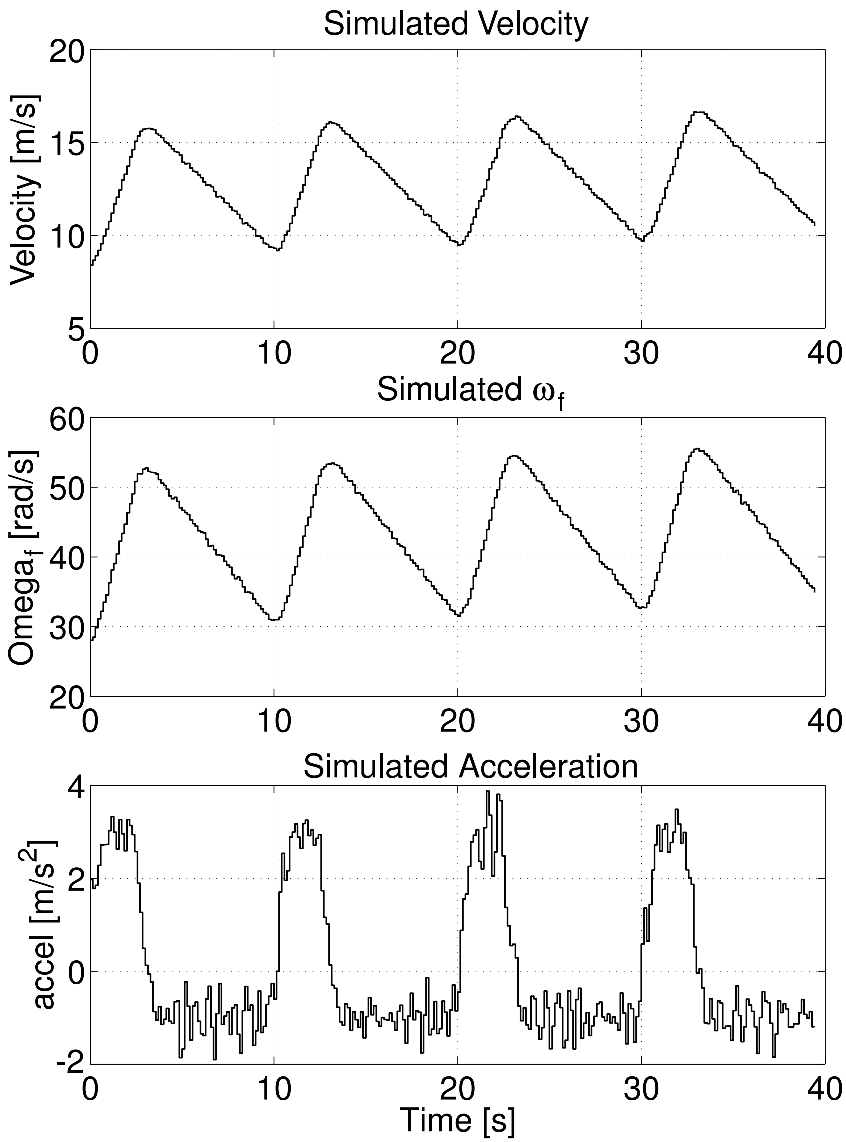
Energy Formulation

Try reformulating problem as energy balance

$$
\begin{aligned}
m\frac{dV}{dt} &= -C_x\left(\frac{V - R\omega}{V}\right) \\
m\int V\,dV &= -C_x\int (V - R\omega)dt \\
mV^2 - mV_0^2 &= -2C_x(S - R\theta)
\end{aligned}
$$

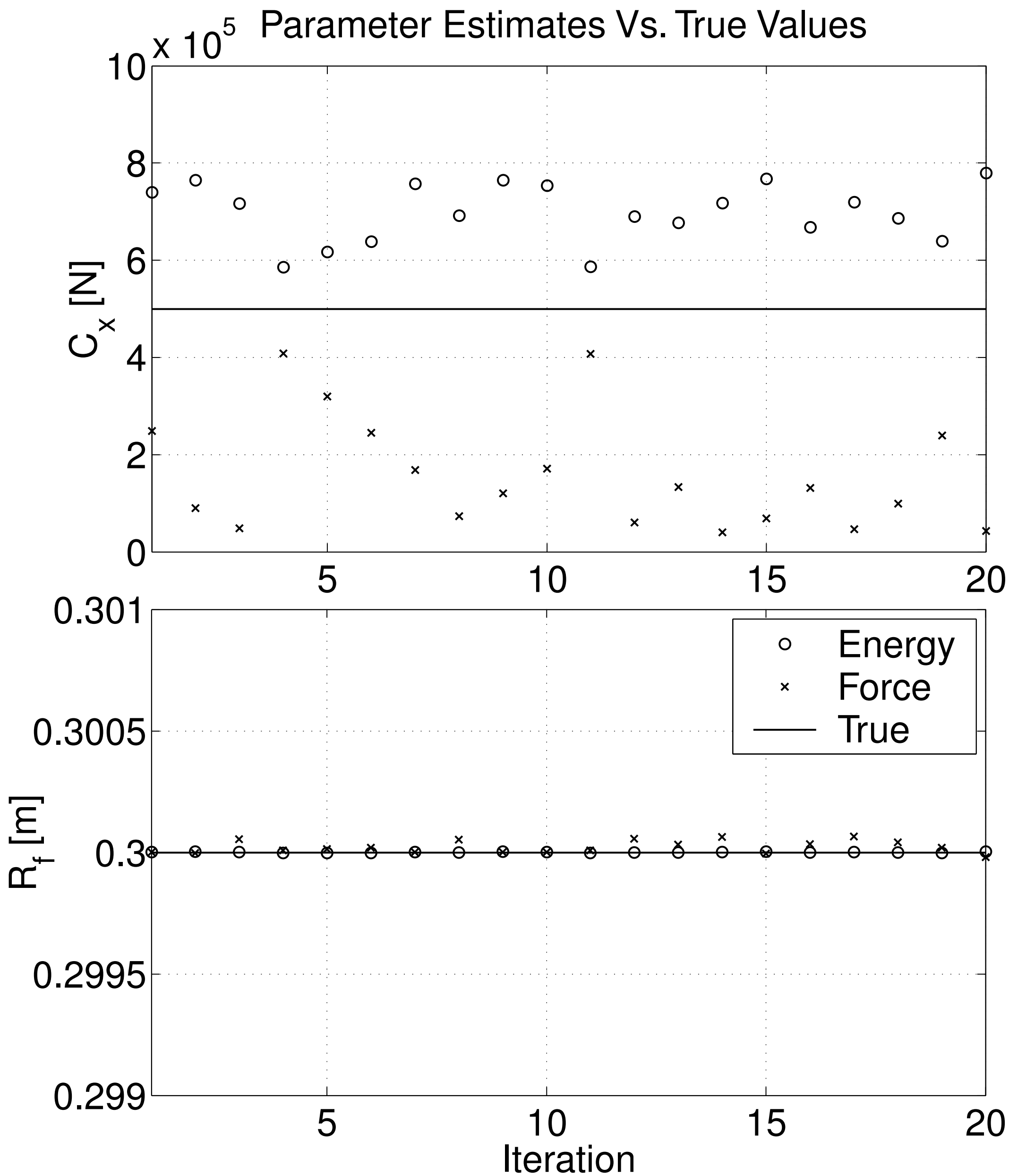
$$
mR_r^2\left(\dot{\hat{\theta}}_r^2 - \dot{\hat{\theta}}_{r0}^2\right) = -2\left[R_r\hat{\theta}_r \;\; -\hat{\theta}_f\right]\begin{bmatrix} C_x \\ R_f C_x \end{bmatrix}
$$

Simulation Setup



Wheel angle measurement noise modeled as white

Simulation Results



Linear least squares estimators return estimates with variance dependent biases

Implicit Regularization

$$\hat{a} = \begin{bmatrix} -\frac{1}{m} & \frac{\hat{\omega}_f}{m\hat{V}} \end{bmatrix} \begin{bmatrix} C_x \\ R_f C_x \end{bmatrix}$$

$$a + \Delta a = \begin{bmatrix} -\frac{1}{m} & \frac{\omega_f + \Delta\omega_f}{m(V + \Delta V)} \end{bmatrix} \begin{bmatrix} C_x \\ R_f C_x \end{bmatrix}$$

Errors in estimation matrix cause bias in parameter estimates

Recast as a Nonlinear Problem

Write out the measurement errors explicitly

$$mR_r(\ddot{\theta_r + \Delta\theta_r}) = -C_x \left(\frac{R_r(\dot{\theta_r + \Delta\theta_r}) - R_f(\dot{\theta_f + \Delta\theta_f})}{R_r(\dot{\theta_r + \Delta\theta_r})} \right)$$

$$\Rightarrow$$

$$f(\hat{\theta}_f, \hat{\theta}_r, \Delta\theta_f, \Delta\theta_r, R_r, C_x) = 0$$

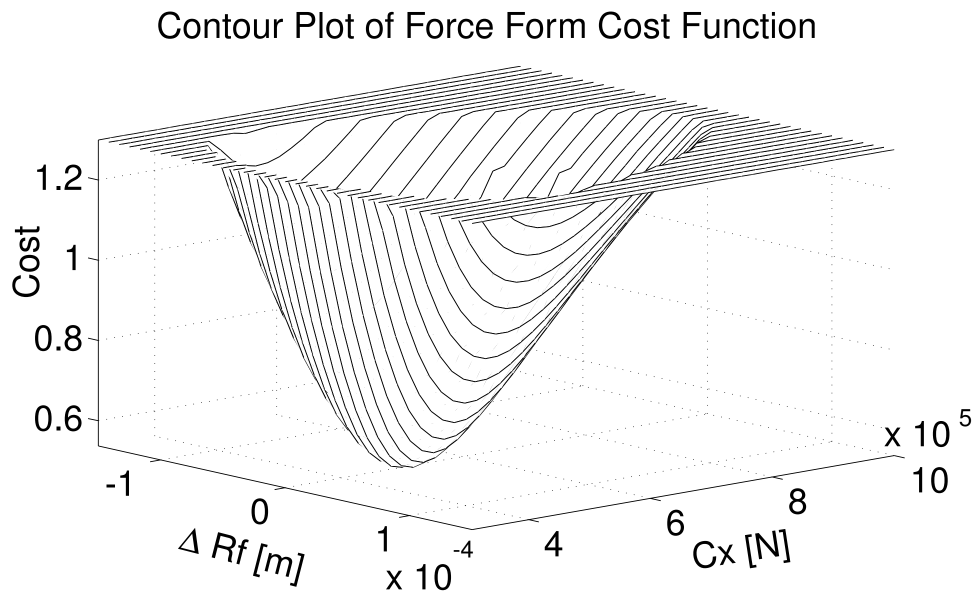
Nonlinear Optimization

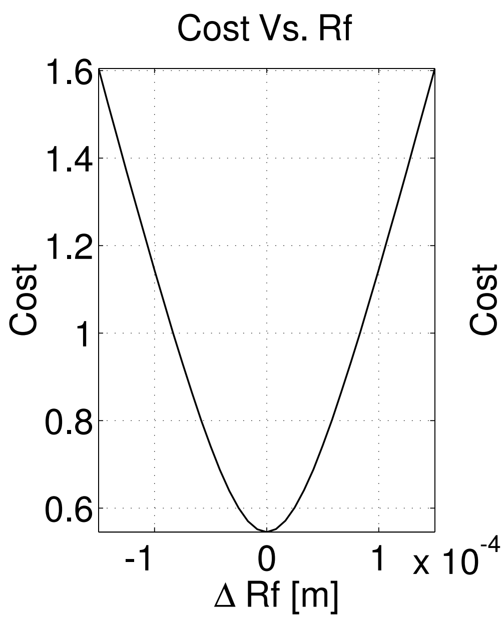
Minimize the measurement errors subject to the force-slip constraint function (orthogonal regression)

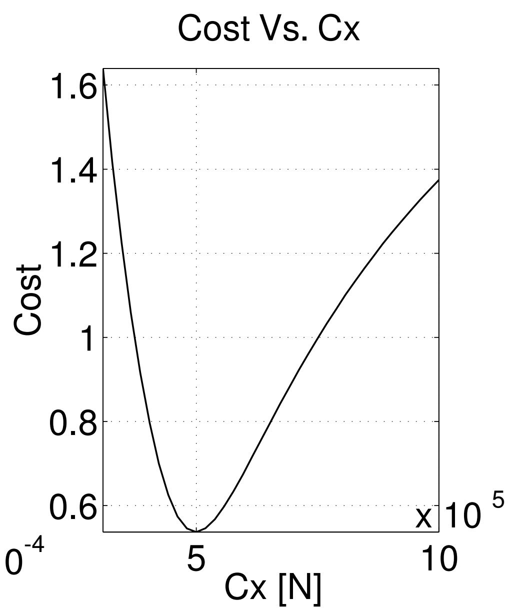
$$\underset{R_r,\, C_x}{\text{Minimize:}} \quad \left\| \begin{matrix} \Delta\theta_f \\ \Delta\theta_r \end{matrix} \right\|$$

$$\text{Subject to:} \quad f(\hat{\theta}_f, \hat{\theta}_r, \Delta\theta_f, \Delta\theta_r, R_r, C_x) = 0$$

Cost Surfaces are Quasiconvex

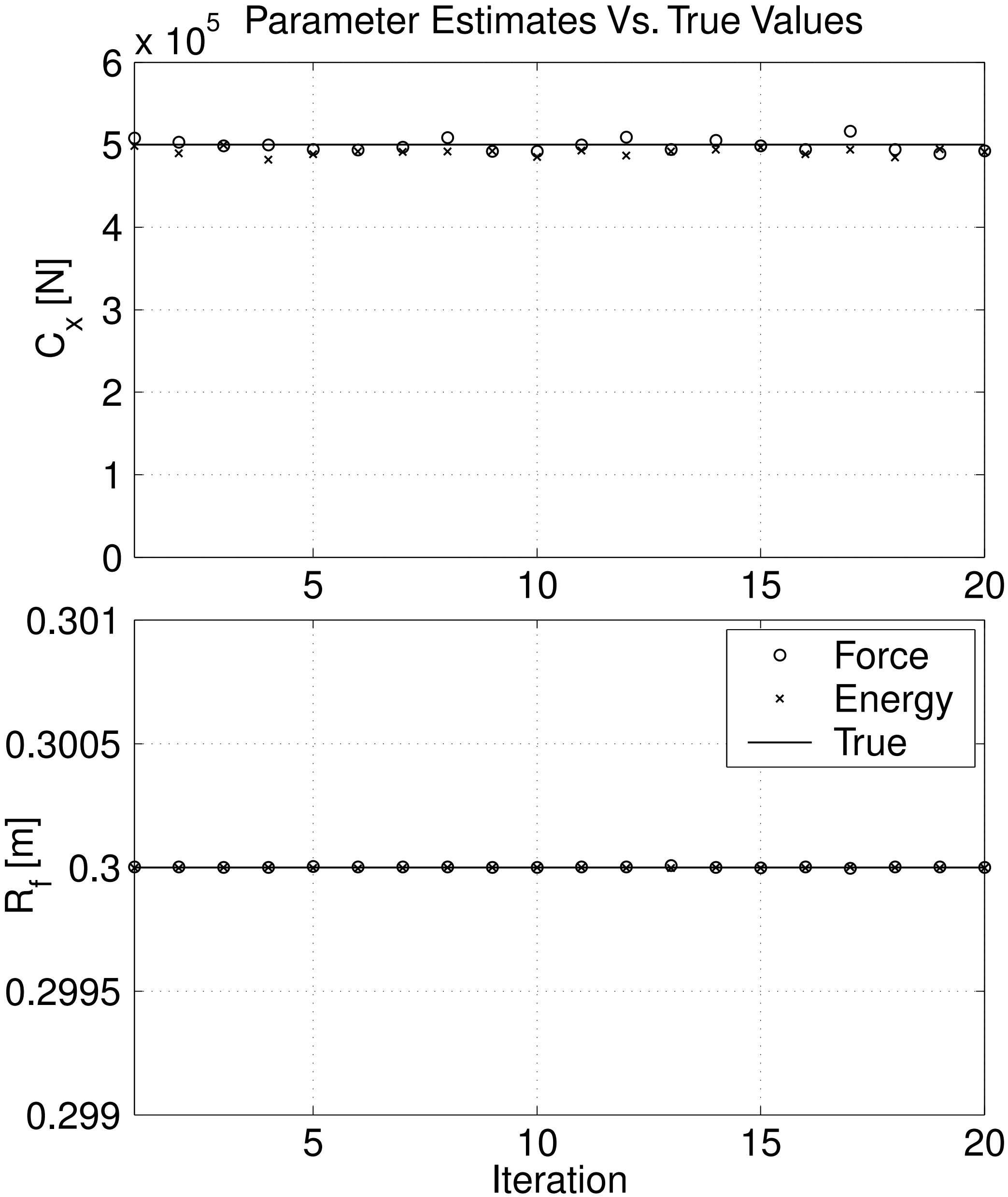






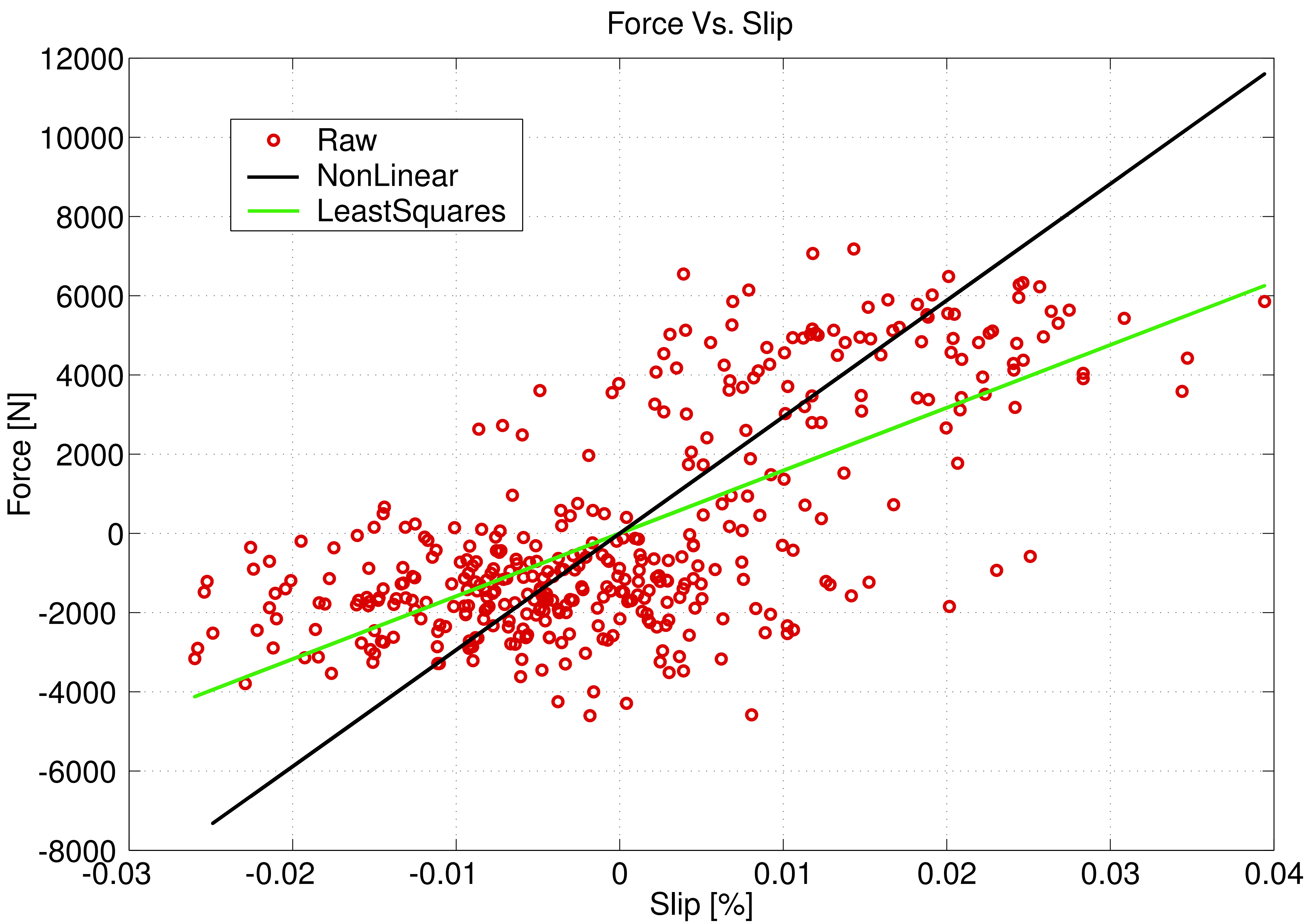
Locally quasiconvex cost function insures unique solution for parameter estimates

Simulation Results

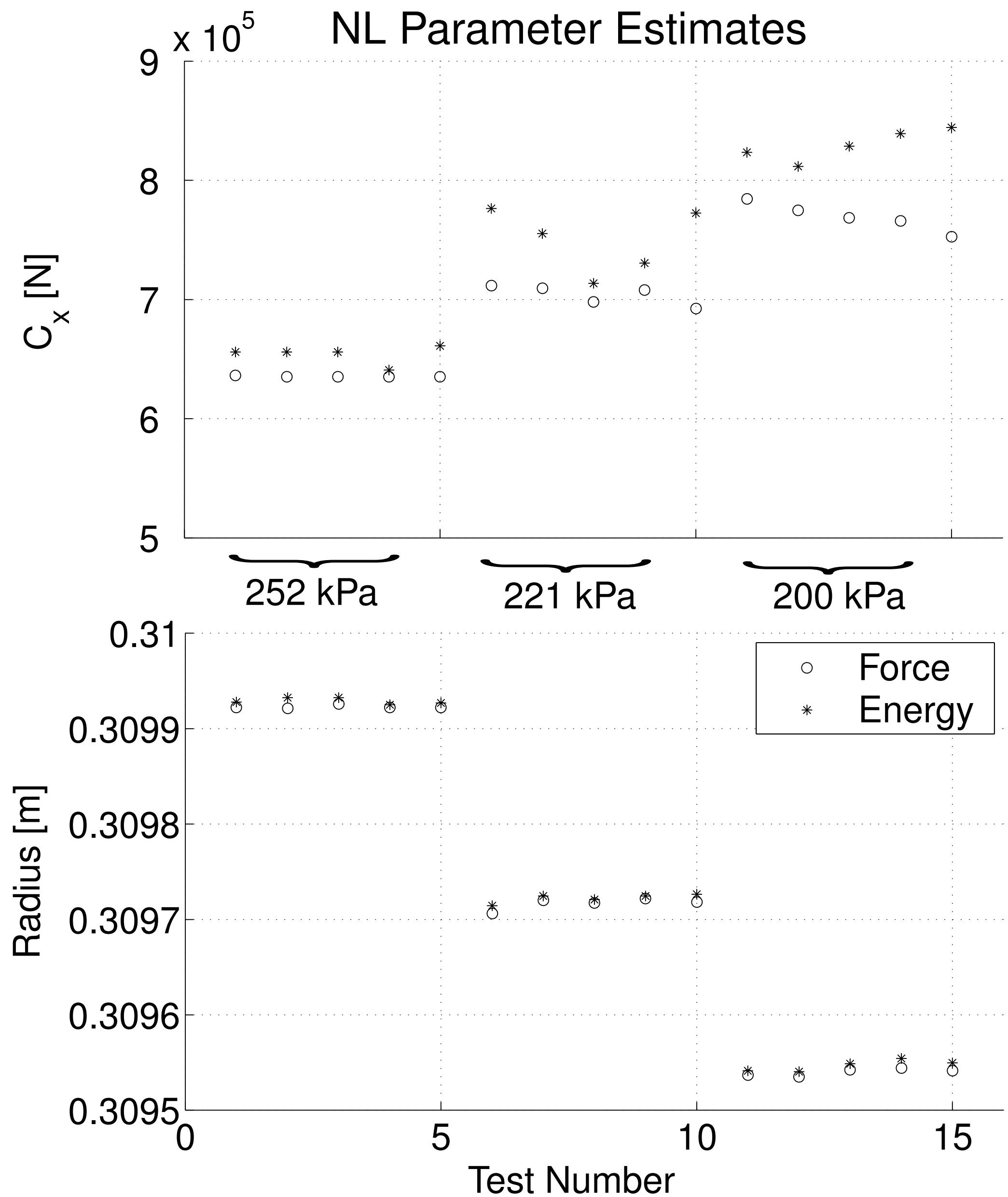


Nonlinear schemes return much more consistent parameter estimates

Simulation Results 2



Test Data



Test data for three different tire pressures.

Each data point represents 60-90 seconds of data

Test Data

- Longitudinal Stiffness estimates repeatable to about 5%
- Strong dependence of C_x upon pressure
- Very small change in effective radius, 0.4 [mm] for 50 [kPa]
- Unmodeled factors may account for errors
  - Road grade
  - Normal load
- Other systematic errors may be present

Conclusions

- □ C_x and R_e sensitive to a number of factors
- □ Estimating tire properties more difficult than it first appears
- □ Linear formulations are biased
- □ Nonlinear schemes appear to work well
- □ longitudinal stiffness inversely proportional to inflation pressure

Future Work

□ Perform experiments on different tires

□ Identify C_x sensitivity to and compensate for

- Tread depth
- Normal load
- Peak road friction

□ Improve algorithm efficiency by several orders of magnitude

□ Combined sideslip, longitudinal slip parameter estimation

Questions ?