A Lyapunov Function Approach to Energy Based Model Reduction

Menu

- Why model reduction?
- Lyapunov functions
- Mass spring damper example
- Tie into Sam’s talk

Model Reduction

- Automated model reduction techniques create complex models
- Engineers are lazy
- Heuristic model reduction requires experience, and
- It is difficult or impossible to quantify the reduction tradeoffs

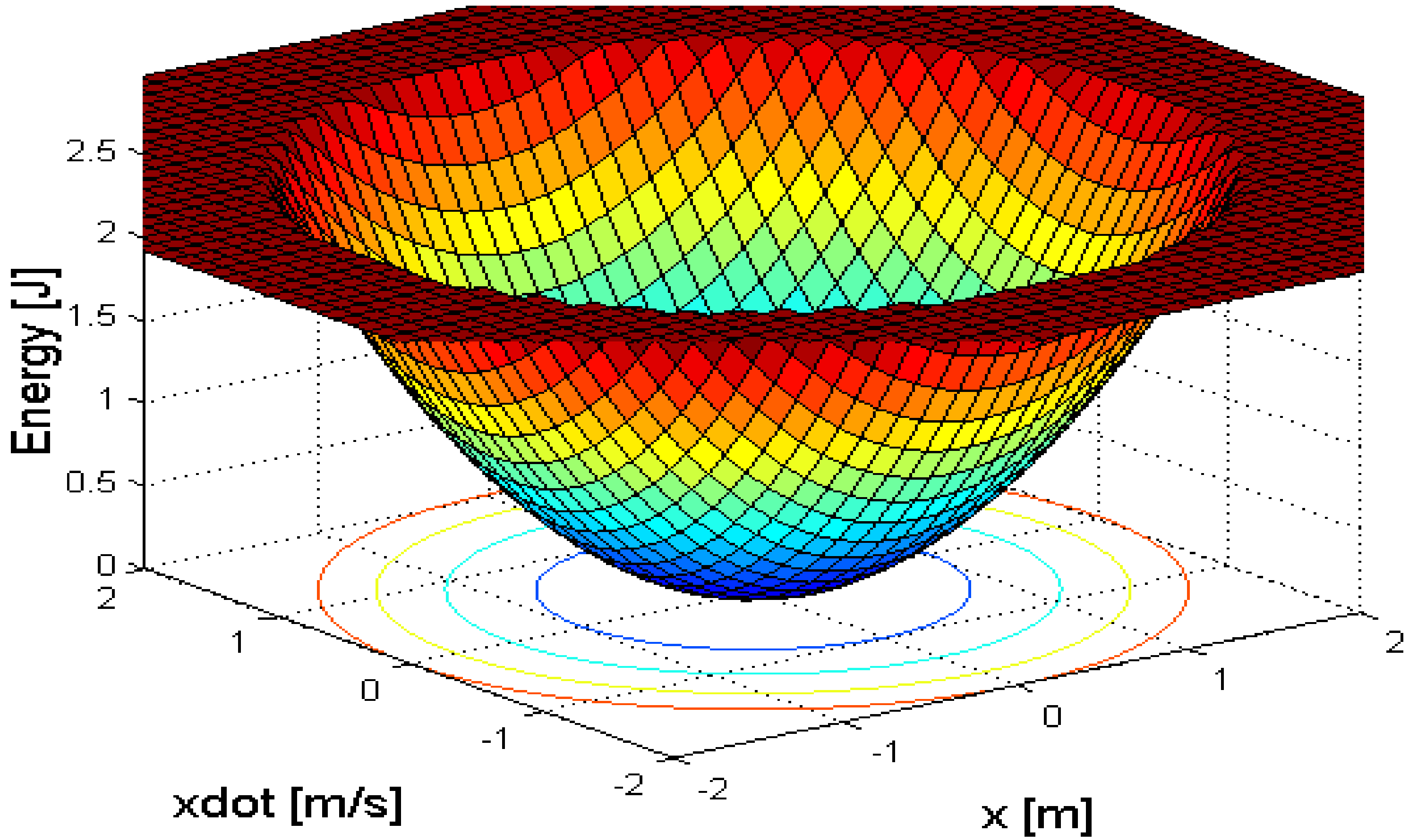
Lyapunov Functions

- Are our friends
- Laypunov (Complicated System) = Scaler Value

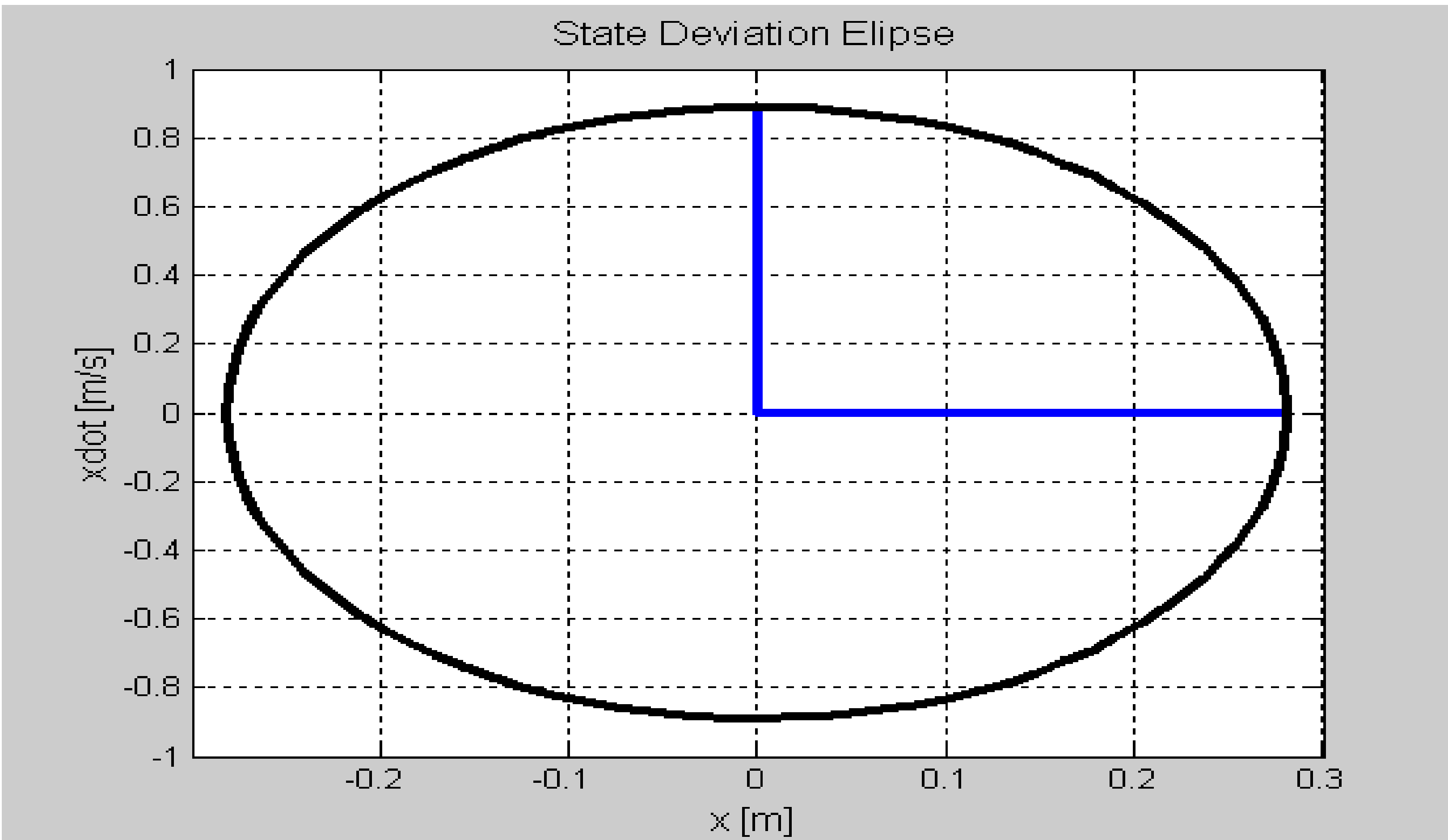
$$f(\bullet):\Re^k \rightarrow \Re$$

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$

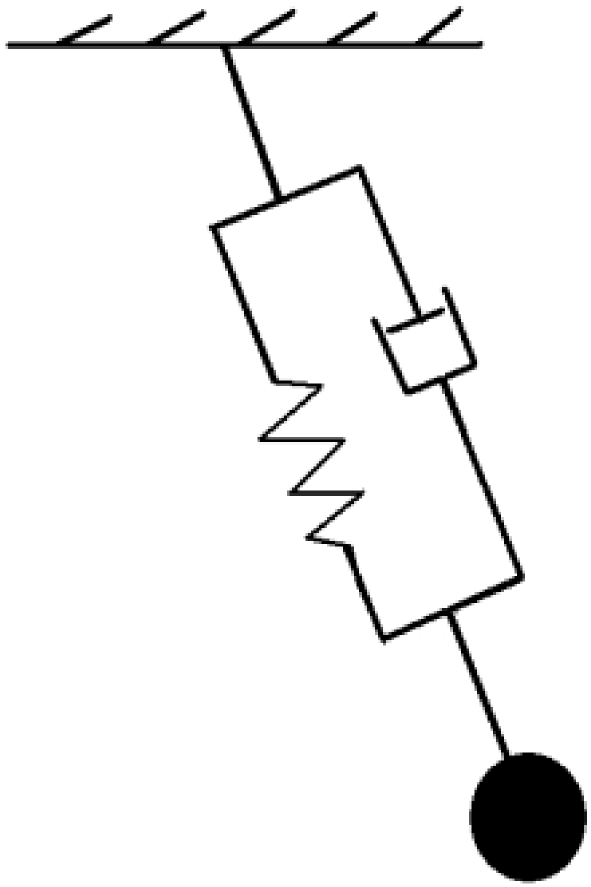
Lyapunov Functions Are Bowls



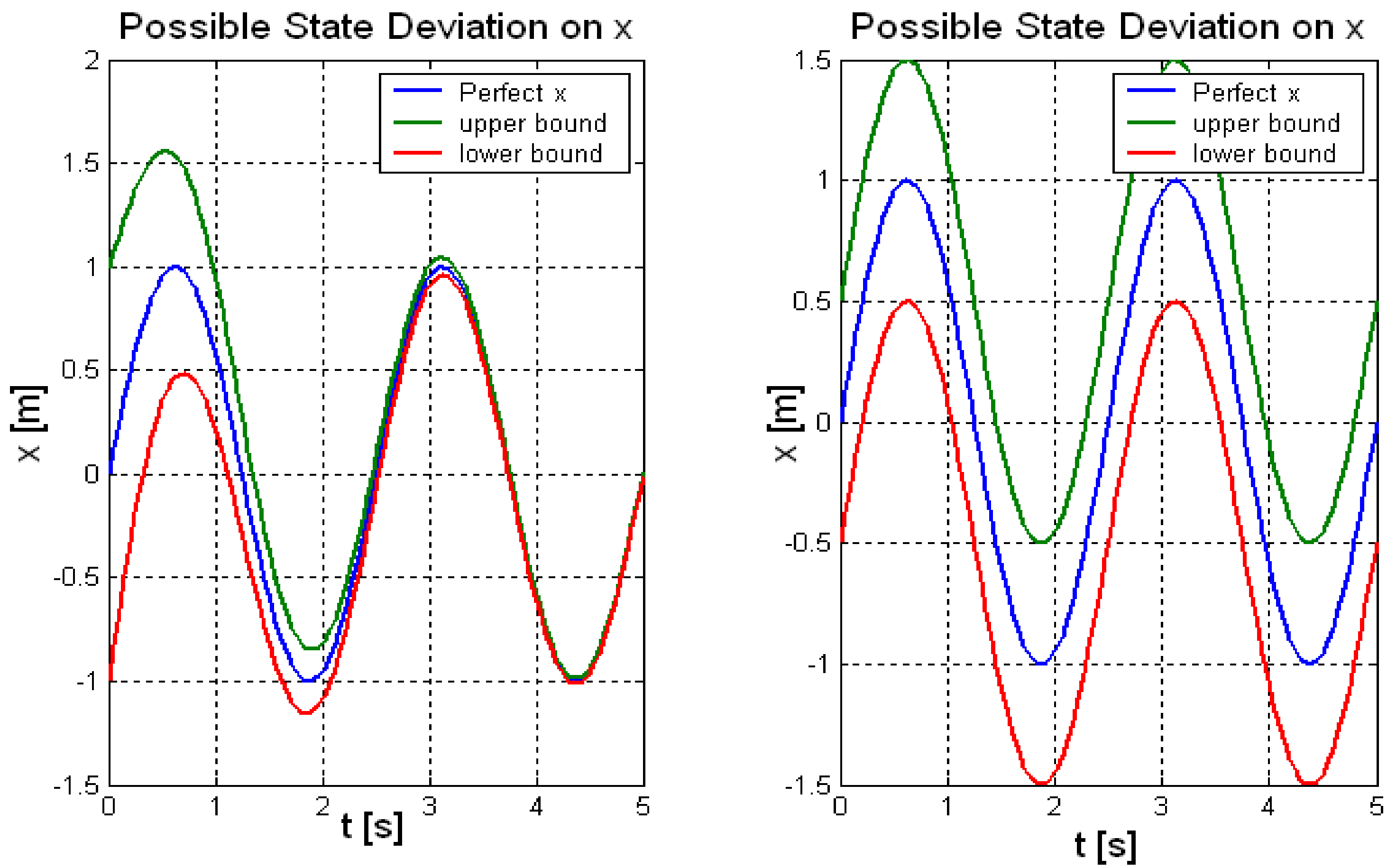
Vector Norms



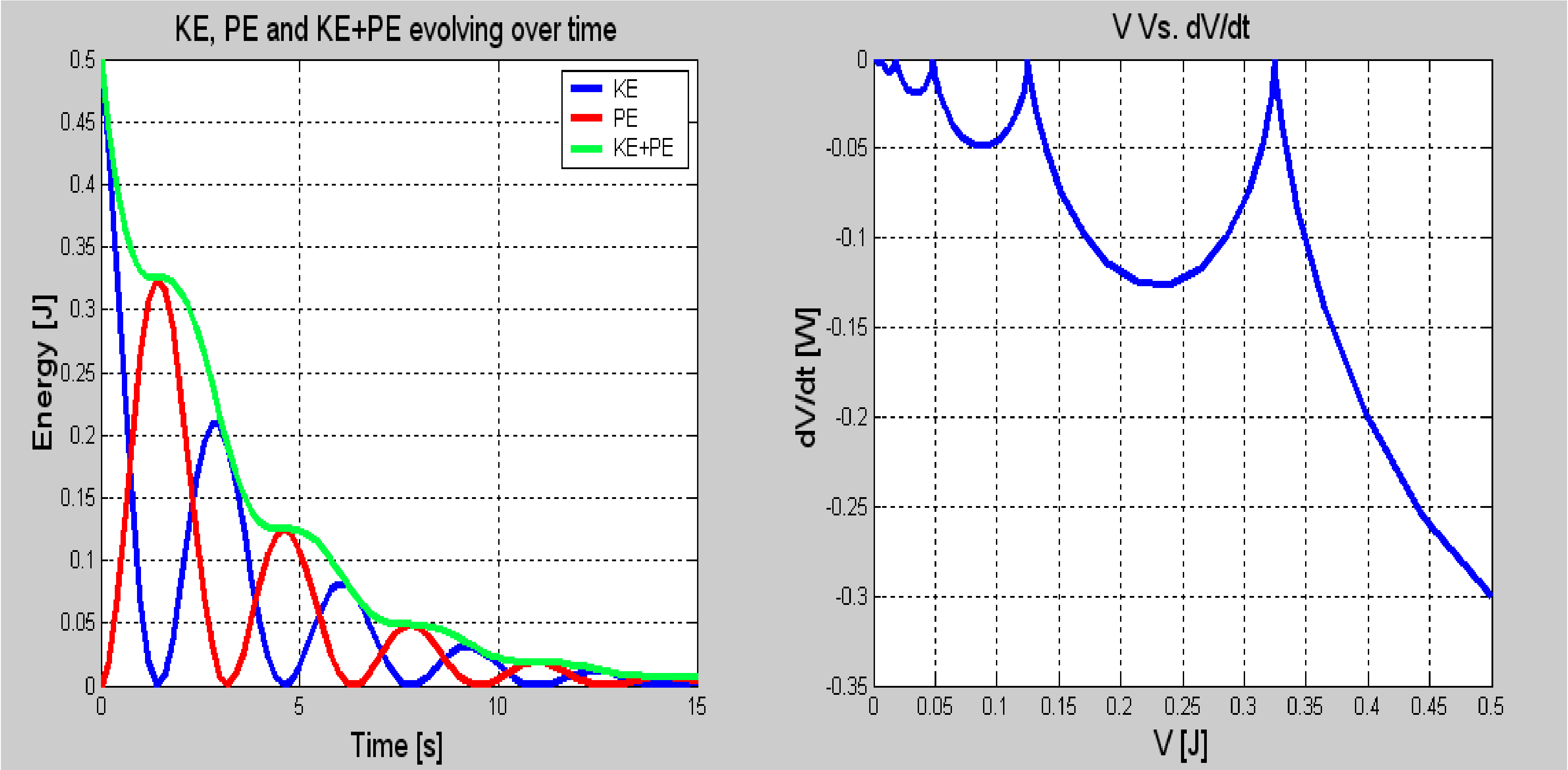
Canonical Example



Pendulum Example



Problem: Negative Semidefinite



Some Math

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \begin{bmatrix} x & \dot{x} \end{bmatrix} \begin{bmatrix} k & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\dot{E} = kx\dot{x} + m\dot{x}\ddot{x} = -b\dot{x} = \begin{bmatrix} x & \dot{x} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

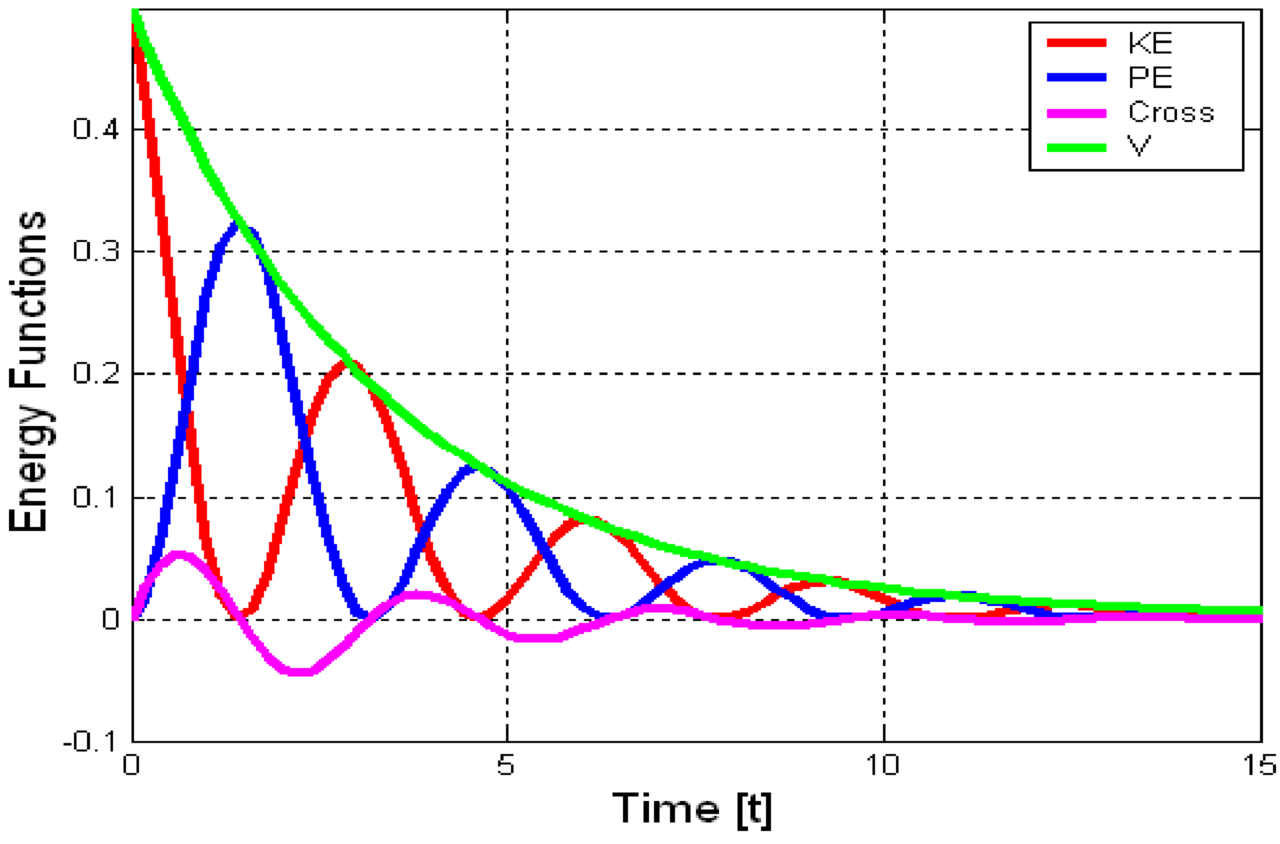
Converse Lyapunov: Look for an Exponential

$$W = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 + \varepsilon x\dot{x}$$

$$\varepsilon = \frac{b}{2} \qquad \alpha = \frac{b}{m}$$

$$\dot{W} = -\alpha W$$

“Mostly” Energy Function



Nonlinear Systems

- Nonlinear systems are more difficult to analyze
- Is there a physical generalization from linear to nonlinear?

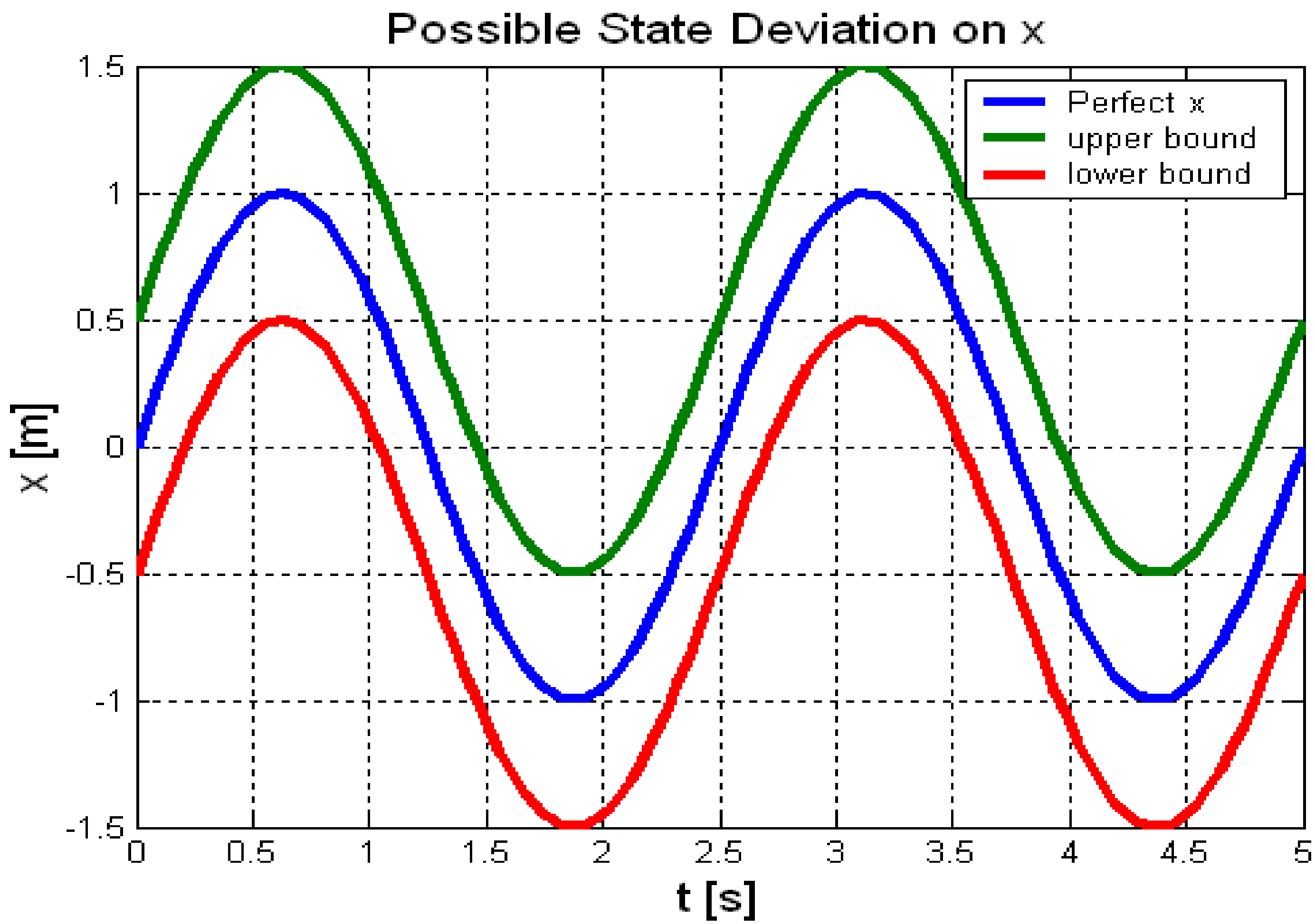
Sam's Talk

- Quantifies the reduction for damped mechanical systems

=> Bounded Force, Bounded State

- This generates a constant ellipsoid of possible state deviation from the simplified model

Bounded Input, Bounded State



Recap & Conclusion

- Automated models are too complex
- Lyapunov functionals are our friends
- Energy is not an ideal Lyapunov function, but it can be “tweaked” in the linear case
- Math is actually cool